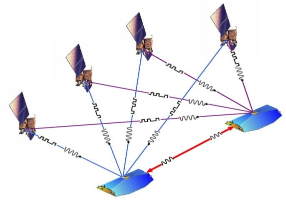

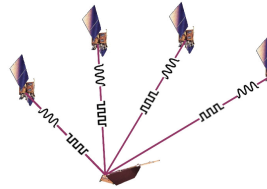

# Kinematical LEO Orbit Determination with sequential time differenced GPS SST carrier phase observations

**Akbar Shabanloui**

**EGU 2007**

**Session GNSS (G6)**

**Wien, Österreich**

**16 April 2007**

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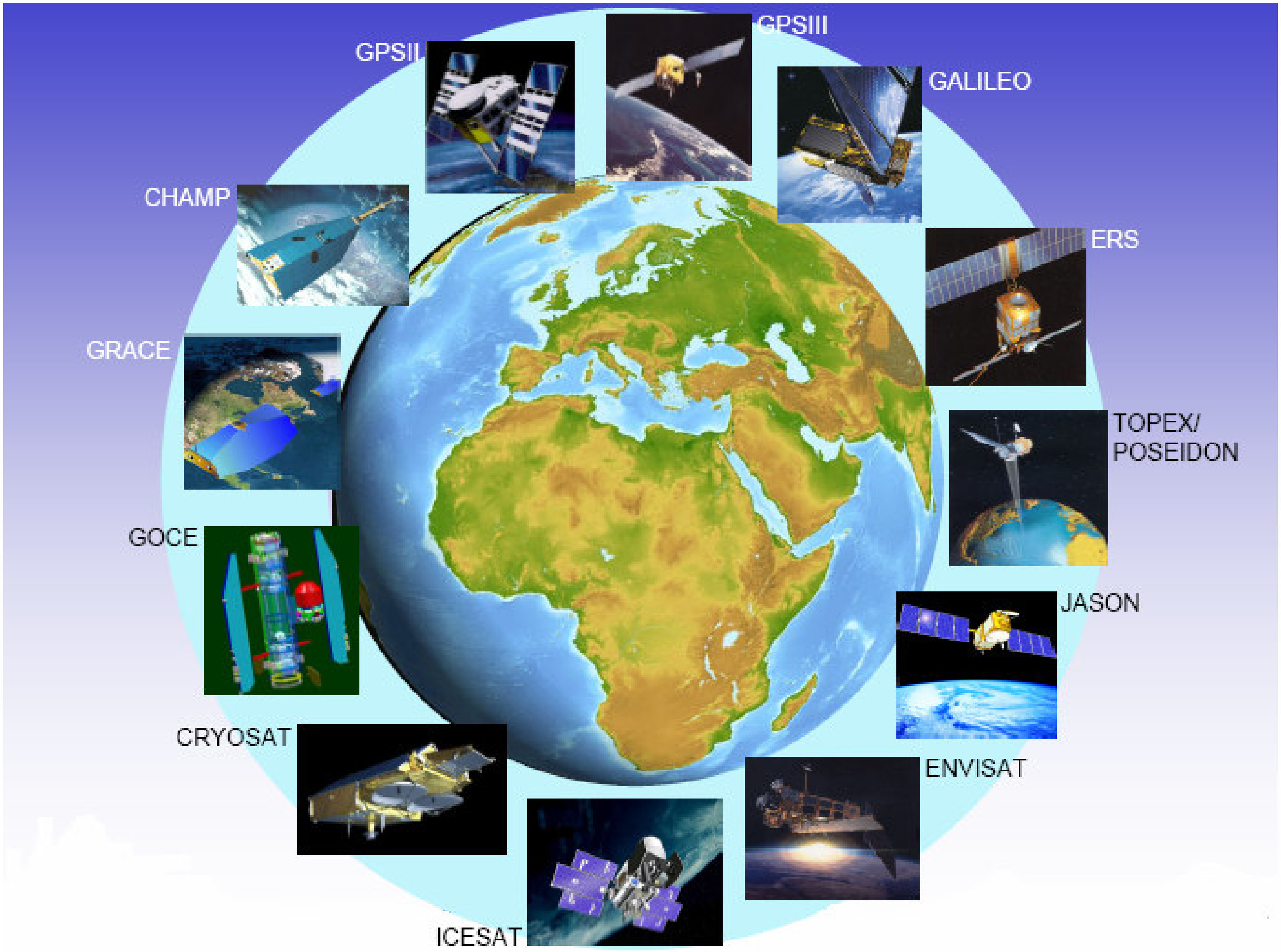
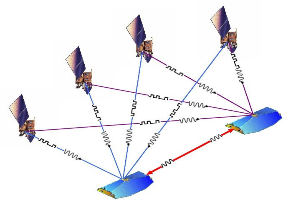
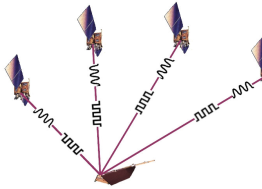
# LEO Missions (Earth Explorers)





# Advantages of LEO Precise Orbit Determination (POD)



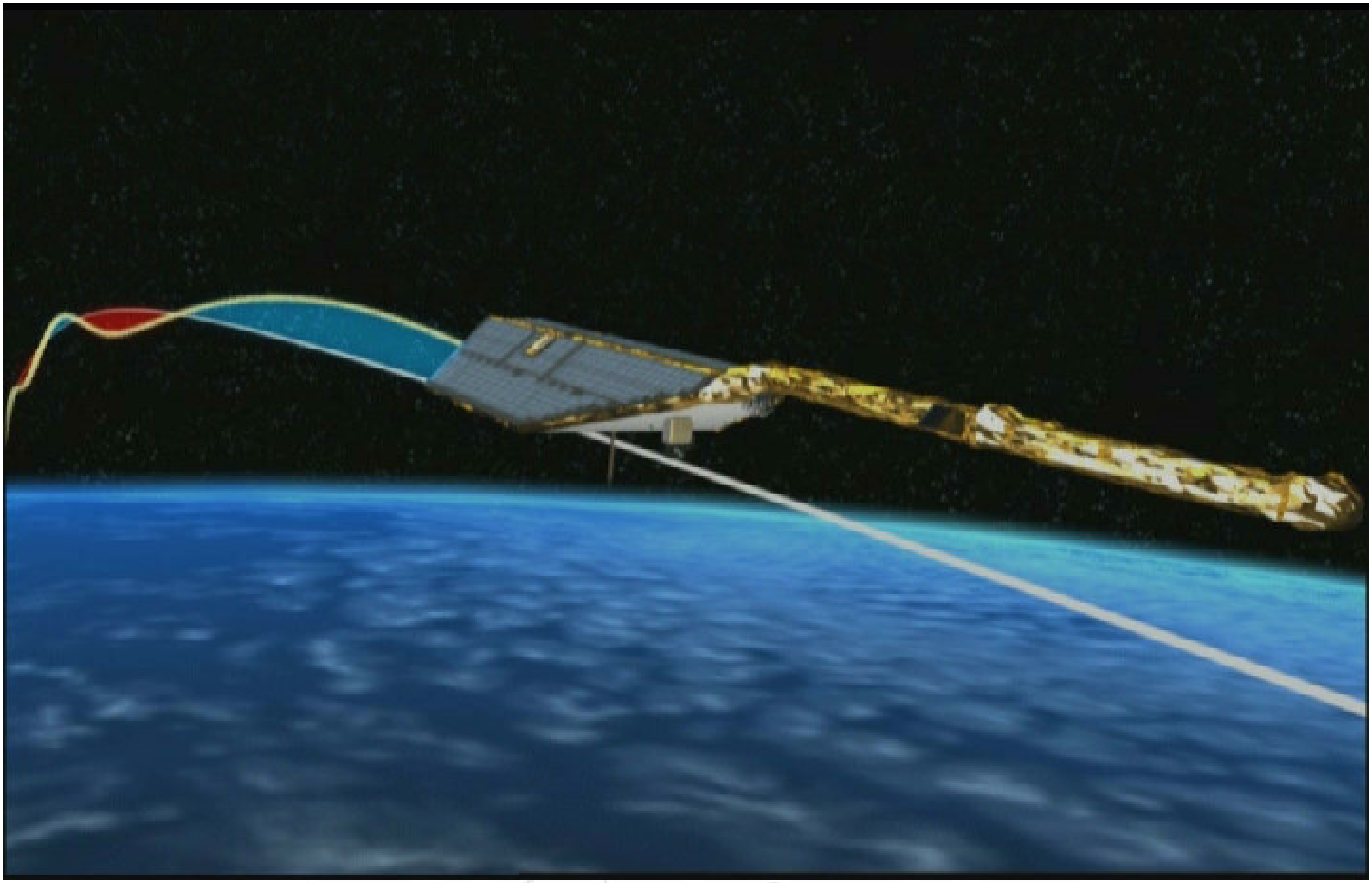
✓Precise LEO orbits can be used to recover the gravity field of the Earth by the POD method

✓Analysis of altimetry observations requires precise orbits

✓Atmosphere sounding requires precise positions of the LEO satellites

✓GNSS (GPS, GLONASS, GALILEO) methods play an important role in POD in addition to classical methods (e.g. SLR…)

# Precise Orbit Determination (POD)



# Principal techniques of POD

✓Geometrical orbit determination

Only geometrical observations are used, no force models and no constraints; pointwise representation

✓Kinematical orbit determination

Geometrical, kinematical observations are used, no force models used; representation by kinematical functions

✓Dynamical orbit determination

Geometrical, kinematical & dynamical observations are used, force models necessary; pointwise representation or representation by functions

# Geometrical precise orbit determination

Observations:

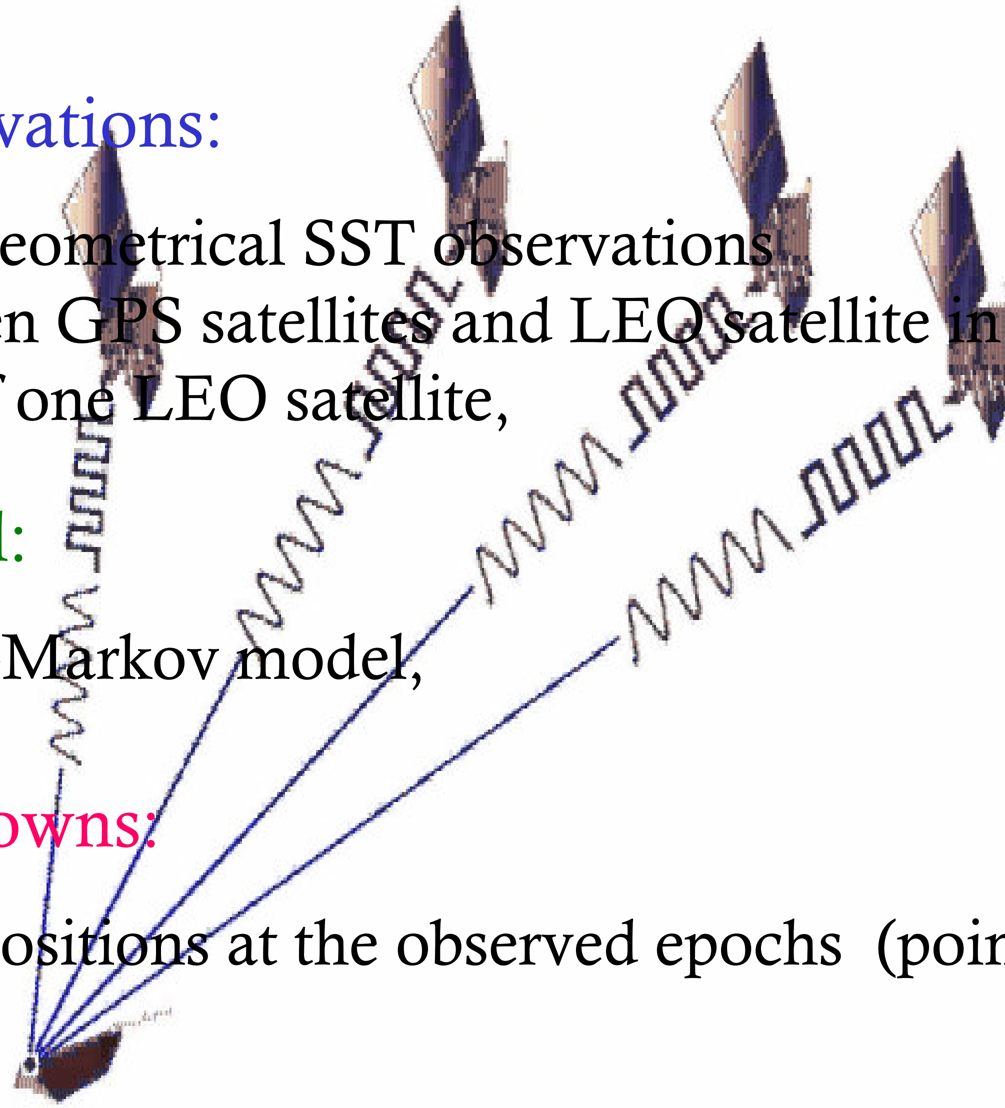
Only geometrical SST observations between GPS satellites and LEO satellite in case of one LEO satellite,

Model:

Gauss-Markov model,

Unknowns:

LEO positions at the observed epochs (point wise).



# Kinematical precise orbit determination

## Observations:

Geometrical SST observations between GPS satellites and LEO satellite in case of one LEO satellite, LEO representation model with respect to physical background

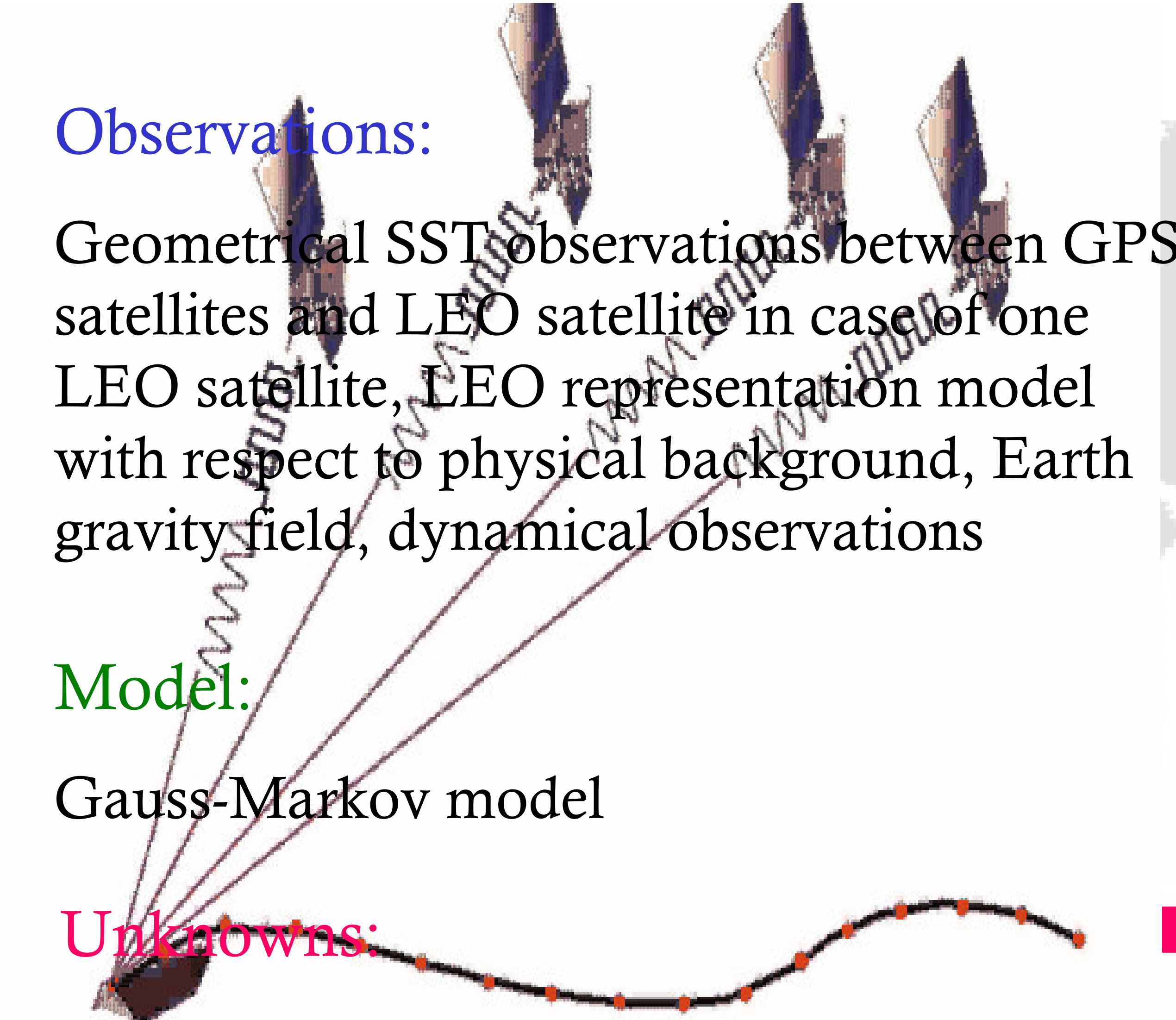
## Model:

Gauss-Markov model,

## Unknowns:

LEO short arc representation parameters (continuous)

# Dynamical precise orbit determination



## Observations:

Geometrical SST observations between GPS satellites and LEO satellite in case of one LEO satellite, LEO representation model with respect to physical background, Earth gravity field, dynamical observations

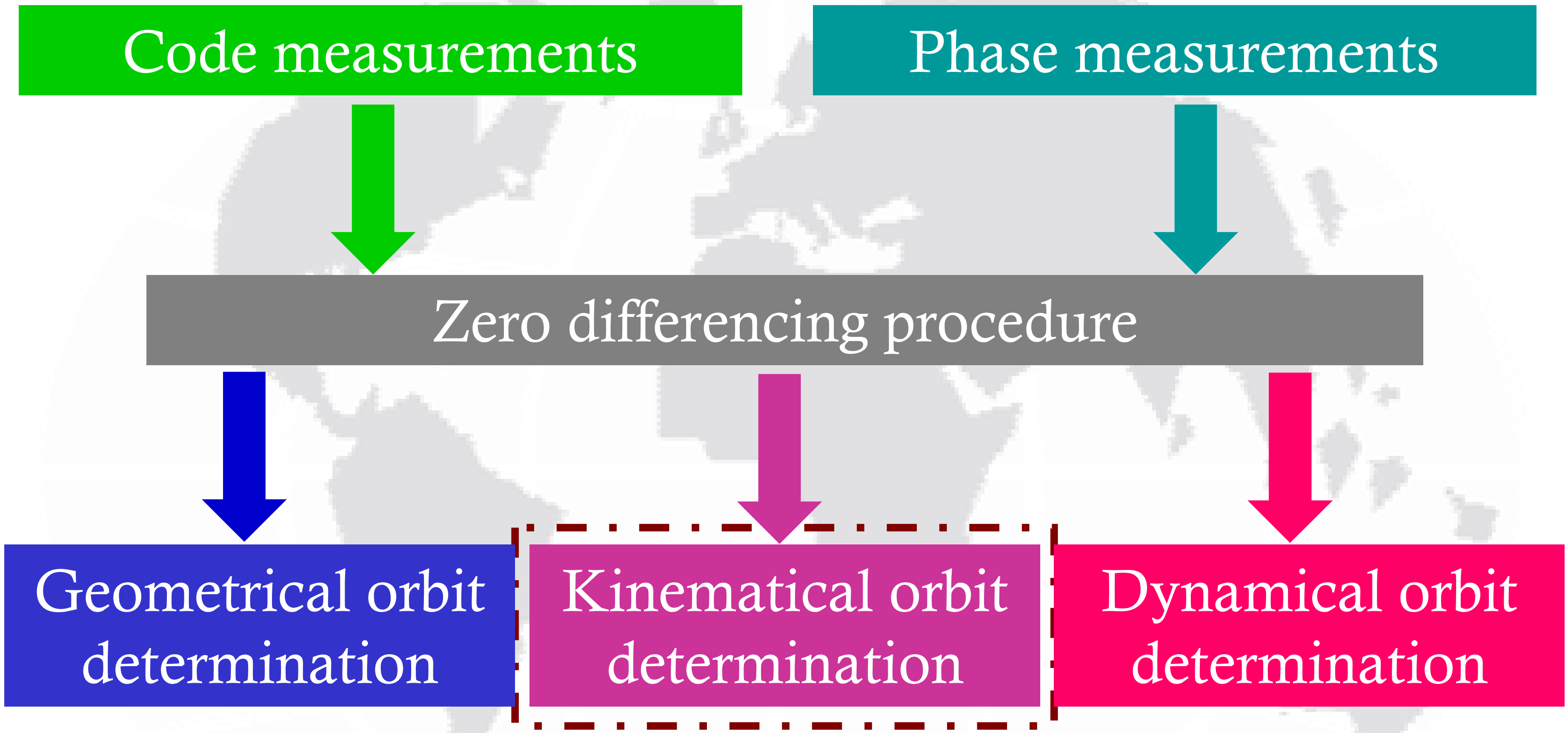
## Model:

Gauss-Markov model

## Unknowns:

LEO boundary positions, (continuous)

# Processing concepts

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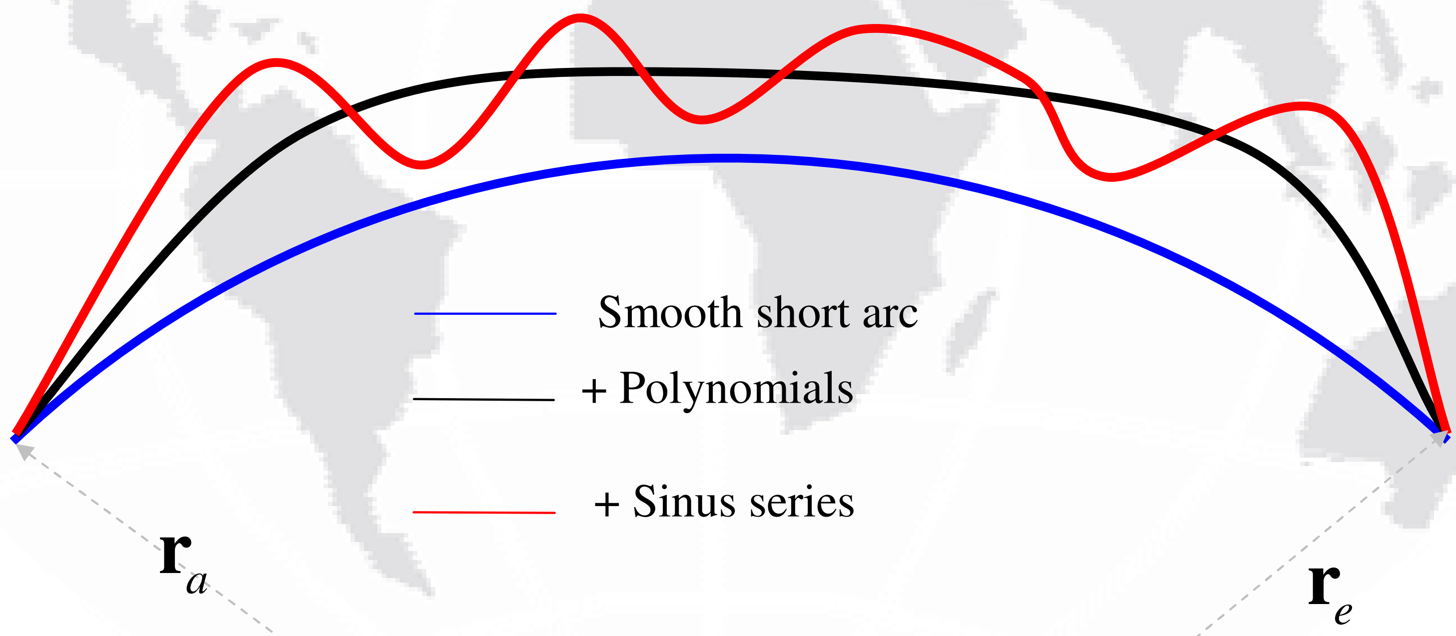
# Kinematical (short arcs) POD concept

# LEO short arc POD principle

## Step by step presentation of short arcs of LEO

$$\mathbf{r} = \underbrace{\mathbf{r}_a \cdot \frac{\sin((1-\tau)\cdot N)}{\sin(N)} + \mathbf{r}_e \cdot \frac{\sin(\tau N)}{\sin(N)}}_{\text{Smooth short arc}} + \underbrace{\mathbf{C}^T\mathbf{P}(\tau)}_{\text{Euler-Bernoulli Polynomial}} + \underbrace{\sum_{\nu=1}^{n} \bar{\bar{\mathbf{d}}}_\nu \sin(\nu\pi\tau)}_{\text{Sinus series}}$$




**representation of LEO short arc**

# Kinematical short arc POD-simulation

selected a-priori

$$\mathbf{r}(t) = \mathbf{r}_a \frac{\sin((1-\tau)N)}{\sin(N)} + \mathbf{r}_e \frac{\sin(\tau N)}{\sin(N)} + \mathbf{C}^T \mathbf{P}(\tau) + \sum_{\nu=1}^{n} \bar{\bar{\mathbf{d}}}_\nu \sin(\nu\pi\tau)$$

**all coefficients can be estimated by a Gauss-Markov model**

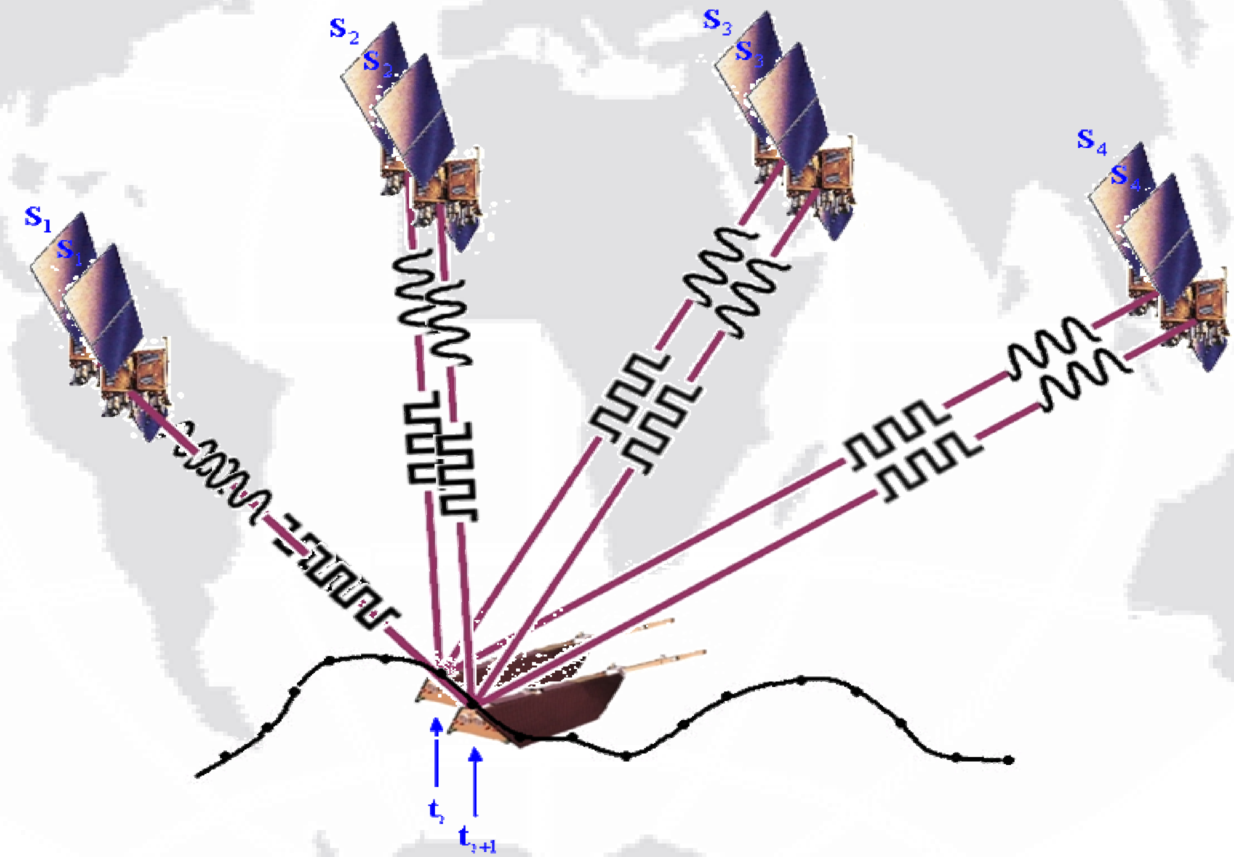
## Advantage:

**the kinematical orbits and another kinematical parameters can be derived directly from estimated LEO short arc parameters.**

# Kinematical (short arcs) POD simulated case

# Absolute position from sequential time differenced carrier phase

- cut-off angle 15°
- simple data processing & S/N filtering
- elevation weighting

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**Sequential time differenced carrier phase observations can be written as:**

$$\Delta\Phi_r^s(t_1,t_2) = \left|\mathbf{r}^s(t_2) - \mathbf{r}_r(t_2)\right| - \left|\mathbf{r}^s(t_1) - \mathbf{r}_r(t_1)\right| + e_{\Delta\Phi_3}$$

$$\Delta\Phi_{r,3}^s(t_1,t_2) =$$

$$\left|\mathbf{r}^s(t_2) - (\mathbf{r}_a.\frac{\sin((1-\tau_2)N)}{\sin(N)} + \mathbf{r}_e.\frac{\sin(\tau_2 N)}{\sin(N)} + \mathbf{C}_{3\times 4}\mathbf{P}(\tau_2) + \sum_{f=1}^{n}\bar{\mathbf{d}}_f.\sin(\pi f \tau_2))\right| -$$

$$\left|\mathbf{r}^s(t_1) - (\mathbf{r}_a.\frac{\sin((1-\tau_1)N)}{\sin(N)} + \mathbf{r}_e.\frac{\sin(\tau_1 N)}{\sin(N)} + \mathbf{C}_{3\times 4}\mathbf{P}(\tau_1) + \sum_{f=1}^{n}\bar{\mathbf{d}}_f.\sin(\pi f \tau_1))\right| + e_{\Delta\Phi_3}$$

# The Gauss-Markov model for one epoch,

$$\begin{pmatrix} \Delta\Delta\Phi_r^{s_1}(t_1,t_2) \\ \vdots \\ \Delta\Delta\Phi_r^{s_j}(t_1,t_2) \\ \vdots \\ \Delta\Delta\Phi_r^{s_{m_1}}(t_1,t_2) \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{\mathbf{r}_r}^{s_1}(t_2)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_2) - \mathbf{A}_{\mathbf{r}_r}^{s_1}(t_1)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_1) \\ \vdots \\ \mathbf{A}_{\mathbf{r}_r}^{s_j}(t_2)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_2) - \mathbf{A}_{\mathbf{r}_r}^{s_j}(t_1)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_1) \\ \vdots \\ \mathbf{A}_{\mathbf{r}_r}^{s_{m_1}}(t_2)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_2) - \mathbf{A}_{\mathbf{r}_r}^{s_{m_1}}(t_1)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_1) \end{pmatrix} [\mathbf{X}_r - \mathbf{X}_r^0]$$

$$\mathbf{X}_r = \begin{pmatrix} x_a & y_a & z_a & \ldots & z_e & c_{11} & c_{12} & c_{13} & c_{14} & \ldots & c_{34} & \bar{\bar{d}}_{1,x} & \bar{\bar{d}}_{1,y} & \bar{\bar{d}}_{1,z} & \ldots & \bar{\bar{d}}_{n,z} \end{pmatrix}^{\mathbf{T}}_{6+12+3n}$$

⇩

$$\Delta\Delta\Phi = \mathbf{A}\Delta\mathbf{X}, \qquad \Sigma_{\Delta\Phi}$$

# Kinematical short arc POD sequential time differenced carrier phase

- **Unknows:** boundary values, polynomial coefficients, amplitudes of Fourier series.
- **Solutions:** Gauss-Markov model
- **Convergence & accuracy:** after a few iterations, ~ cm.

✓ LEO short arc orbit is continuous and velocities and another kinematical parameters can be derived from the estimated parameters.

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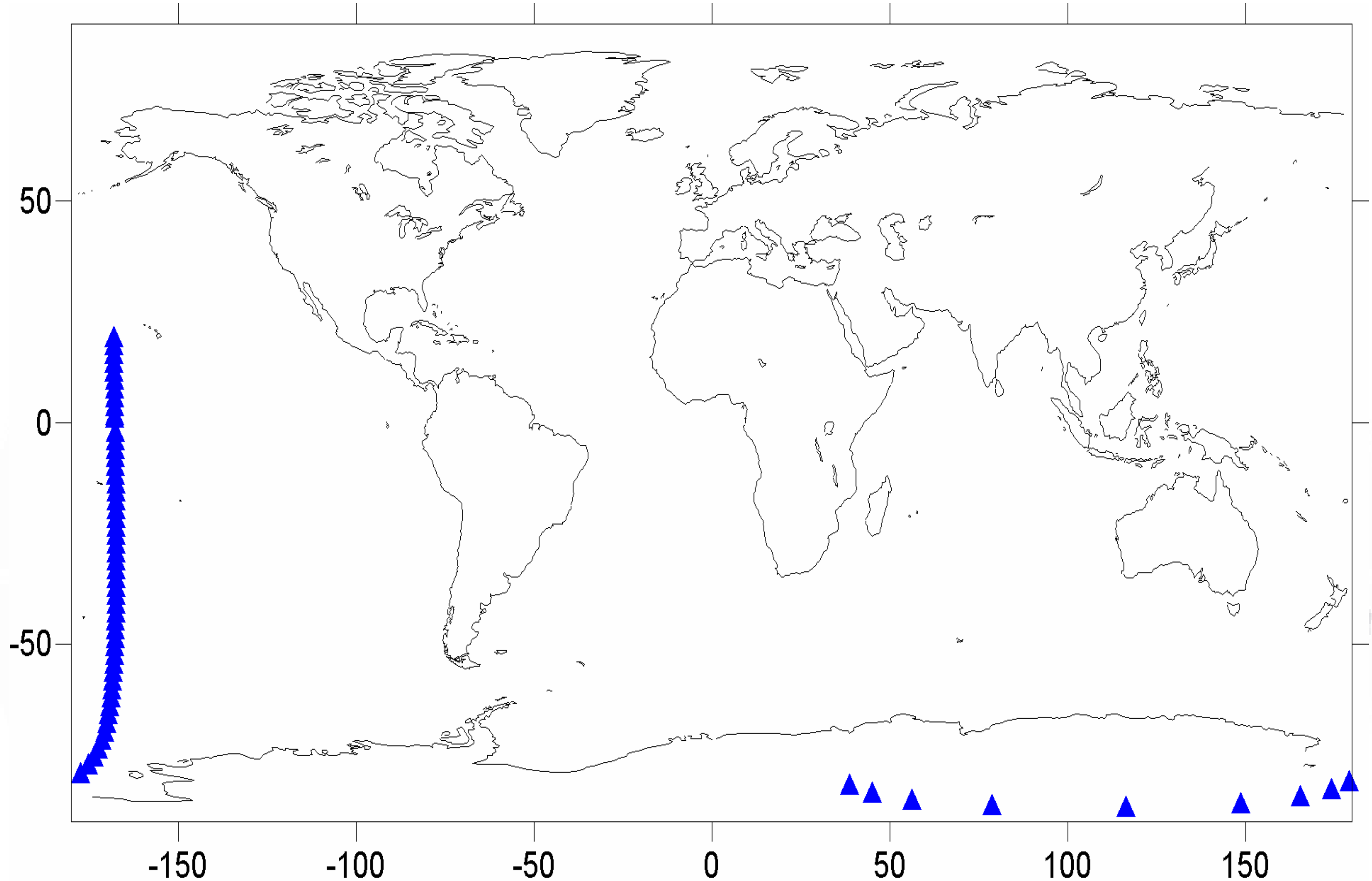
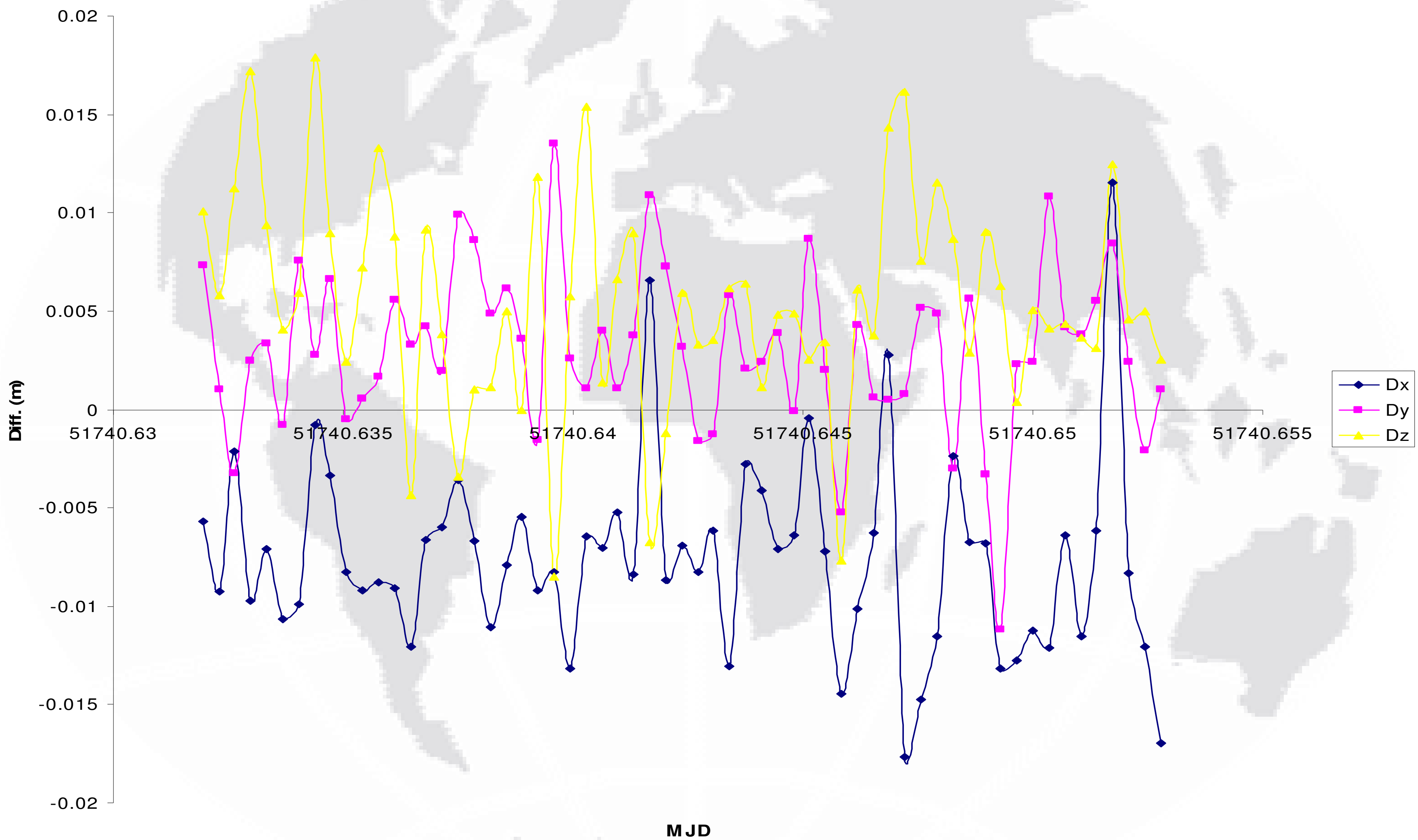
# Kinematical short arc POD-simulated




**30 minutes of CHAMP satellite [ 2000 07 15 15h 10m – 15h 40m ]**

# Kinematical short arc POD sequential time differenced carrier phase

Difference plot between estimated short arc absolute positions with observation precision=0.01 m & given positions

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# Kinematical (short arcs) POD real case

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# Carrier phase GPS SST observation

$$\Phi_{r,i}^{s}(t) = \left| \mathbf{R}_z(\omega_e . \varepsilon_r^s) \mathbf{r}^s (t - \varepsilon_r^s) - \mathbf{r}_r(t) \right| + c\left[ dt^s(t - \varepsilon_r^s) - dt_r(t) \right] + \lambda N_r^s +$$

$$I_i^r(t) + d_O^s(t) + d_R^r(t) - d_R^s(t) + d_{C,i}^r(t) + d_{V,i}^r(t) + d_{M,P_i}(t) + e_{P_i}$$

| | |
|---|---|
| $s, r$ | **GPS, LEO indices,** |
| $\varepsilon_r^s$ | **Travelling time between GPS & LEO,** |
| $\mathbf{r}^s(t - \varepsilon_r^s), dt^s(t - \varepsilon_r^s)$ | **GPS position, clock offset at sending time,** |
| $\mathbf{r}_r(t), dt_r(t)$ | **LEO position, clock offset at receiving time** |

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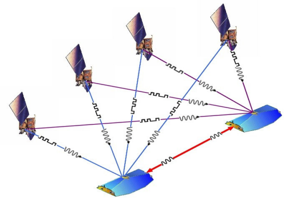
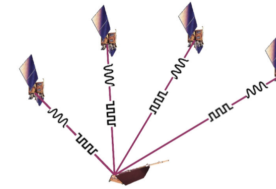


# Carrier phase GPS SST observation…



$I_i^r(t)$

•for single frequency receiver, the IONEX model can be used to model the ionosphere error term,
•for dual frequency receiver, the ionosphere free combination can be used.

$d_O^s(t)$

to … used in

$d_R^s(t), d_R^r($

&

$d_{C,i}^r(t), d_{V,i}^r(t)$

be modeled … s … NTEX file.

How can the errors be eliminated or modeled in GPS LEO SST observations?

$d_{M,P_i}(t)$

multipath effect can be minimized through filtering SST observations w.r.t elevation of GPS satellites or applying the elvation weighting method or S/N filtering.

# Sequential time differenced carrier phase SST

**Carrier phase ionosphere-free observation at epochs (1,2)**

$$\Phi^s_{r,3}(t_1)=\left|\mathbf{R}_z(\omega_e\varepsilon_1)\mathbf{r}^s(t_1-\varepsilon_1)-\mathbf{r}_r(t_1)\right|+\lambda_3 N^s_{r,3}+c\left[dt^s(t_1-\varepsilon_1)-dt_r(t_1)\right]+$$

$$d^s_O(t_1)+d^r_R(t_1)-d^s_R(t_1)+d^r_{C,3}(t_1)+d^r_{V,3}(t_1)+d_{M,\Phi_3}(t_1)+e_{\Phi_3}$$

$$\Phi^s_{r,3}(t_2)=\left|\mathbf{R}_z(\omega_e\varepsilon_2)\mathbf{r}^s(t_2-\varepsilon_2)-\mathbf{r}_r(t_2)\right|+\lambda_3 N^s_{r,3}+c\left[dt^s(t_2-\varepsilon_2)-dt_r(t_2)\right]+$$

$$d^s_O(t_2)+d^r_R(t_2)-d^s_R(t_2)+d^r_{C,3}(t_2)+d^r_{V,3}(t_2)+d_{M,\Phi_3}(t_2)+e_{\Phi_3}$$

**sequential time difference ionosphere-free carrier phase observation between epochs (1,2)**

$$\Delta\Phi^s_{r,3}(t_1,t_2)=\left|\mathbf{R}_z(\omega_e\varepsilon_2)\mathbf{r}^s(t_2-\varepsilon_2)-\mathbf{r}_r(t_2)\right|-\left|\mathbf{R}_z(\omega_e\varepsilon_1)\mathbf{r}^s(t_1-\varepsilon_1)-\mathbf{r}_r(t_1)\right|-$$

$$c\Delta dt_r(t_1,t_2)+e_{\Delta\Phi_3}$$

# Ionosphere free sequential time differenced carrier phase observations can be written as:

$$\Delta\Phi^{s}_{r,3}(t_1,t_2)=\left|\mathbf{R}_z(\omega_e\varepsilon_2).\mathbf{r}^s(t_2-\varepsilon_2)-\mathbf{r}_r(t_2)\right|-\left|\mathbf{R}_z(\omega_e\varepsilon_1).\mathbf{r}^s(t_1-\varepsilon_1)-\mathbf{r}_r(t_1)\right|-$$

$$c\Delta dt_r(t_1,t_2)+e_{\Delta\Phi_3}$$

$$\Delta\Phi^{s}_{r,3}(t_1,t_2)=$$

$$\left|\mathbf{R}_z(\omega_e\varepsilon_2).\mathbf{r}^s(t_2-\varepsilon_2)-(\mathbf{r}_a.\frac{\sin((1-\tau_2)N)}{\sin(N)}+\mathbf{r}_e.\frac{\sin(\tau_2 N)}{\sin(N)}+\mathbf{C}_{3\times4}\mathbf{P}(\tau_2)+\sum_{f=1}^{n}\bar{\bar{\mathbf{d}}}_f\sin(\pi f\tau_2))\right|-$$

$$\left|\mathbf{R}_z(\omega_e\varepsilon_1).\mathbf{r}^s(t_1-\varepsilon_1)-(\mathbf{r}_a.\frac{\sin((1-\tau_1)N)}{\sin(N)}+\mathbf{r}_e.\frac{\sin(\tau_1 N)}{\sin(N)}+\mathbf{C}_{3\times4}\mathbf{P}(\tau_1)+\sum_{f=1}^{n}\bar{\bar{\mathbf{d}}}_f\sin(\pi f\tau_1))\right|-$$

$$c\Delta dt_r(t_1,t_2)+e_{\Delta\Phi_3}$$

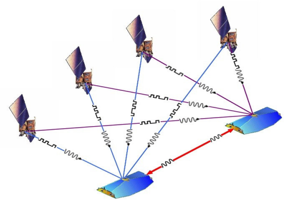
# The Gauss-Markov model for one epoch,

$$\begin{pmatrix} \Delta\Delta\Phi_r^{s_1}(t_1,t_2) \\ \vdots \\ \Delta\Delta\Phi_r^{s_j}(t_1,t_2) \\ \vdots \\ \Delta\Delta\Phi_r^{s_{m_1}}(t_1,t_2) \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{\mathbf{r}_r}^{s_1}(t_2)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_2) - \mathbf{A}_{\mathbf{r}_r}^{s_1}(t_1)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_1) \\ \vdots \\ \mathbf{A}_{\mathbf{r}_r}^{s_j}(t_2)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_2) - \mathbf{A}_{\mathbf{r}_r}^{s_j}(t_1)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_1) \\ \vdots \\ \mathbf{A}_{\mathbf{r}_r}^{s_{m_1}}(t_2)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_2) - \mathbf{A}_{\mathbf{r}_r}^{s_{m_1}}(t_1)\mathbf{A}_{\mathbf{X}_r}^{\mathbf{r}_r}(t_1) \end{pmatrix} [\mathbf{X}_r - \mathbf{X}_r^0] + \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 1 & 0 & 0 \\ \vdots & \ddots & \vdots \\ 1 & \dots & 0 \end{pmatrix} [\mathbf{X}_{\Delta t} - \mathbf{X}_{\Delta t}^0]$$

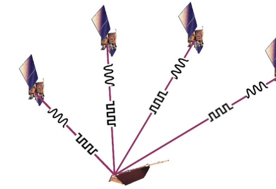
$$\mathbf{X}_r = \begin{pmatrix} x_a & y_a & z_a & \dots & z_e & c_{11} & c_{12} & c_{13} & c_{14} & \dots & c_{34} & \bar{\bar{d}}_{1,x} & \bar{\bar{d}}_{1,y} & \bar{\bar{d}}_{1,z} & \dots & \bar{\bar{d}}_{n,z} \end{pmatrix}^{\mathbf{T}}_{6+12+3n}$$

$$\mathbf{X}_{\Delta t} = (\Delta c\mathrm{dt}_{12} \quad \dots \quad \Delta c\mathrm{dt}_{(\mathrm{m}-1)\mathrm{m}})^{\mathbf{T}}_{m-1}$$

$$\mathbf{\Delta\Delta\Phi} = \mathbf{A}\Delta\mathbf{X}, \qquad \Sigma_{\mathbf{\Delta\Phi}}$$



# Kinematical short arc POD sequential time differenced carrier phase

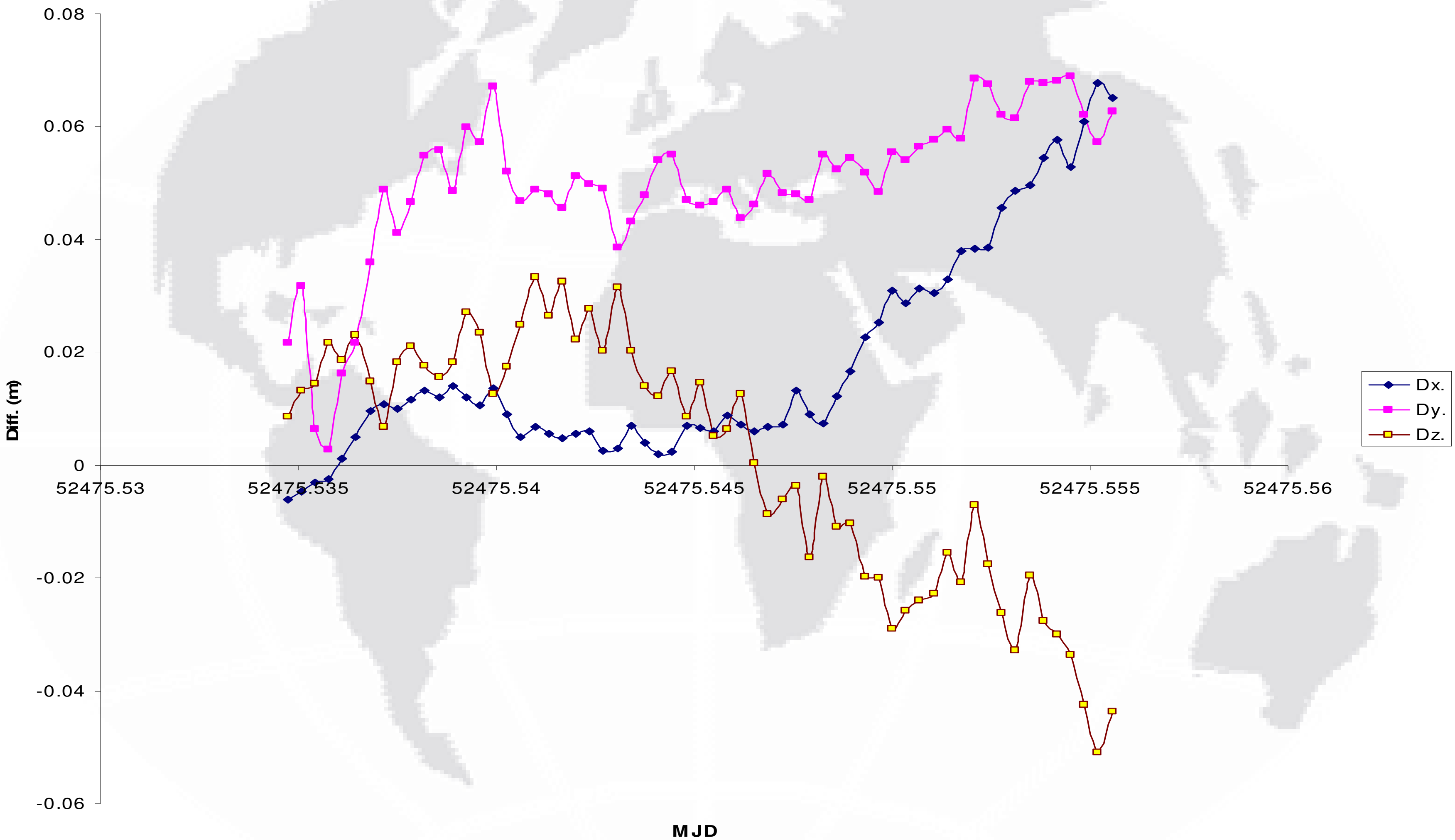


- **Initial values:** unknowns initial values can be derived from code estimated positions at first step,
- **Unknows:** boundary values, polynomial coefficients, amplitudes of Fourier series,
- **Solutions:** Gauss-Markov model,
- **Convergence & accuracy:** after ~a few iterations, ~ cm.

✓ LEO short arc orbit is continuous and velocities and another kinematical parameters can be derived from the estimated LEO short arc parameters.

# Kinematical short arc POD with sequential time differenced carrier phase

Difference plot between absolute positions with carrier phase observation precision =0.01 m & given GFZ CHAMP PSO orbit

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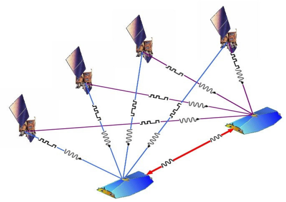
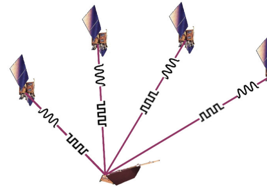
# Conclusions & remarks (kinematical)

- From sequential differenced carrier phase SST observations, LEO absolute positions and clock offsets can be estimated at every epoch with (or even without) enough number of GPS satellites (4?) and good satellite geometry
- An accuracy of cm can be expected for the sequnetial time diffenced carrier phase SST data processing, but DOP! isn‘t crucial
- The resulting LEO orbit is given continuous (without gaps) & smoother than geometrical POD
- Kinematical LEO orbit can be used to recover the Earth‘s gravity field with the POD recovery concept. (kinematical parameters can be derived analytically)





# Thank you for your attentions