From pure kinematical to reduced kinematical LEO orbit determination

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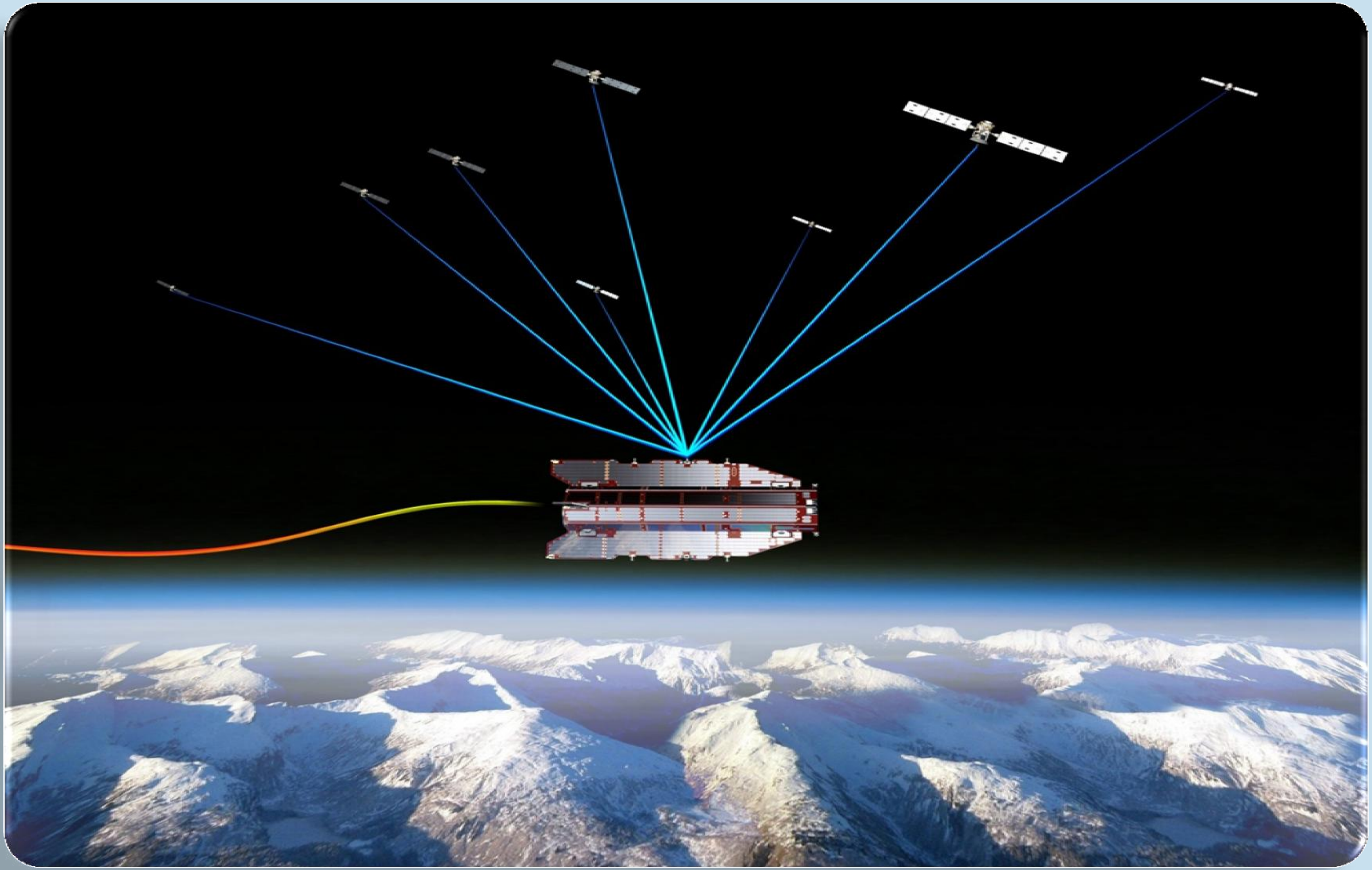
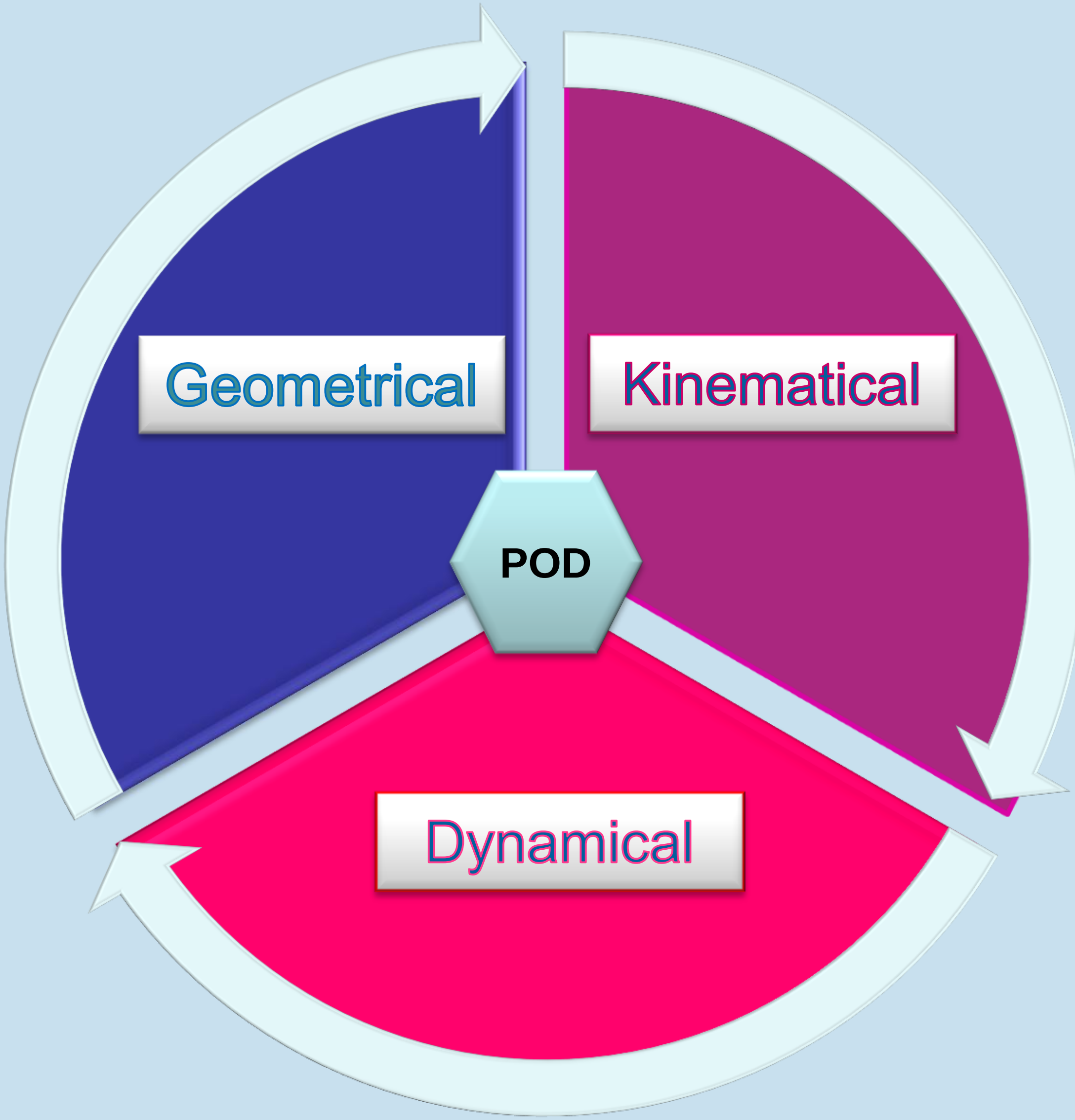
Precise Orbit Determination (POD)

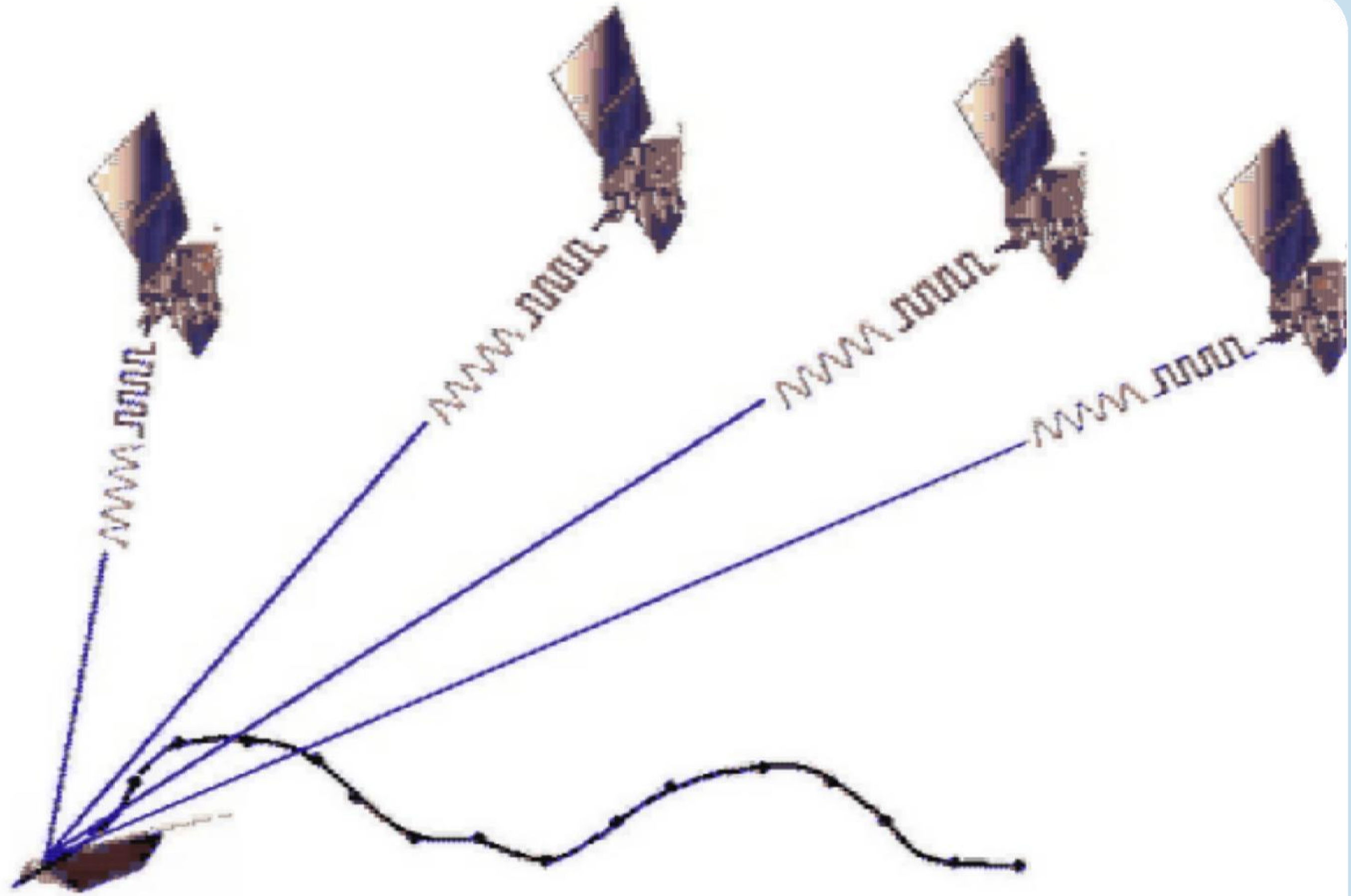


Credit: European Space Agency

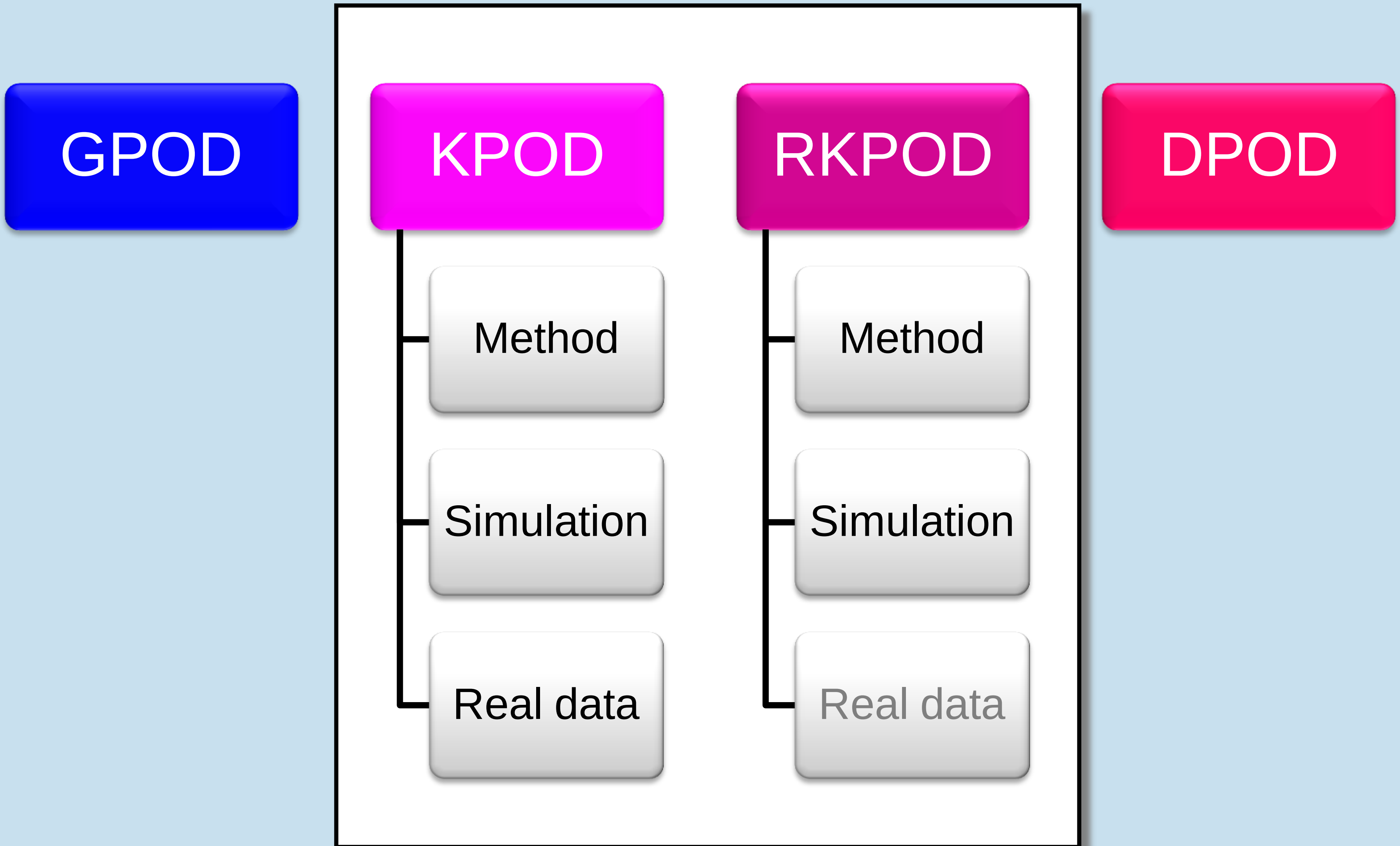
Precise Orbit Determination (POD) methods



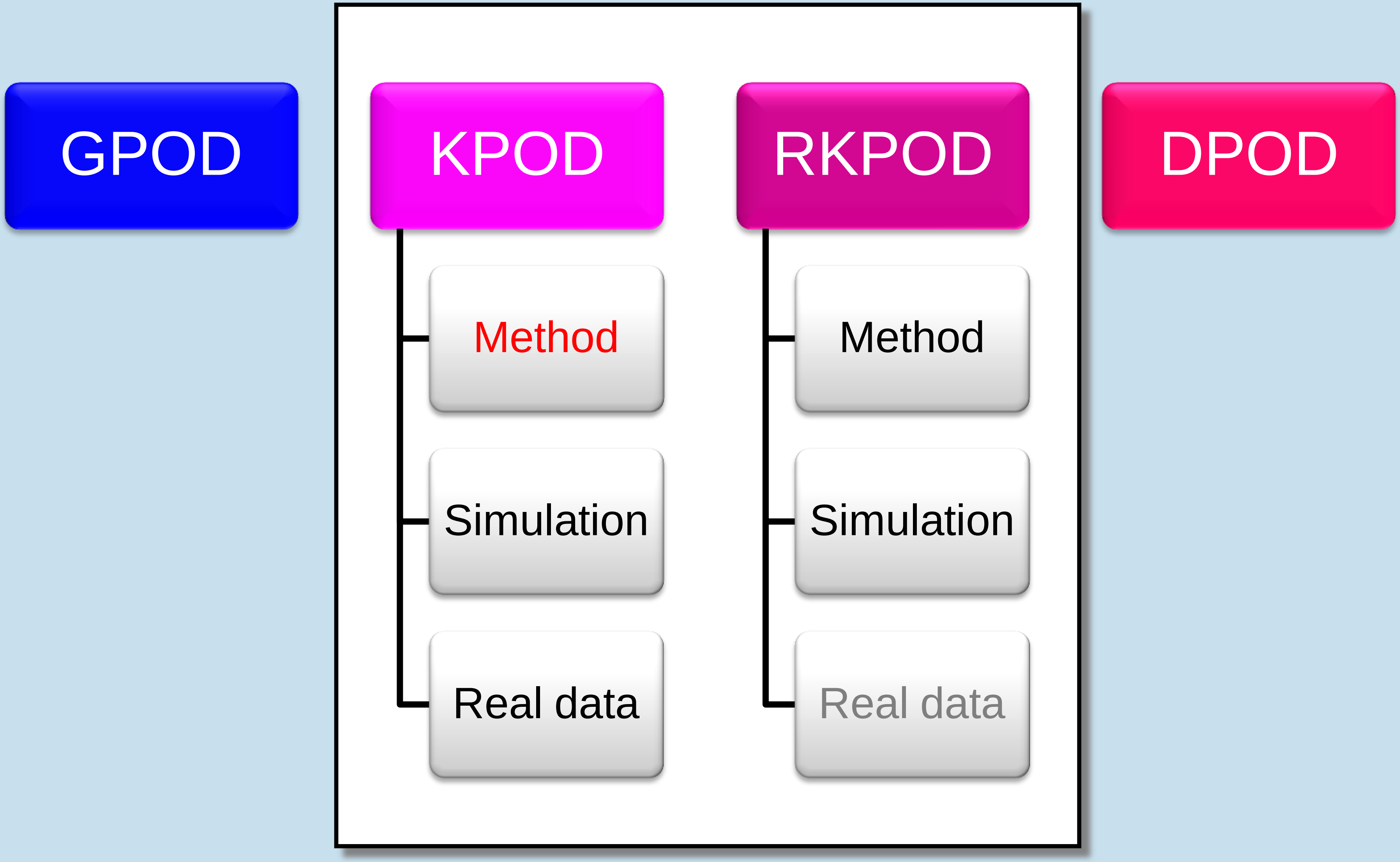
Kinematical Precise Orbit Determination (KPOD)



Processing procedures



Processing procedures

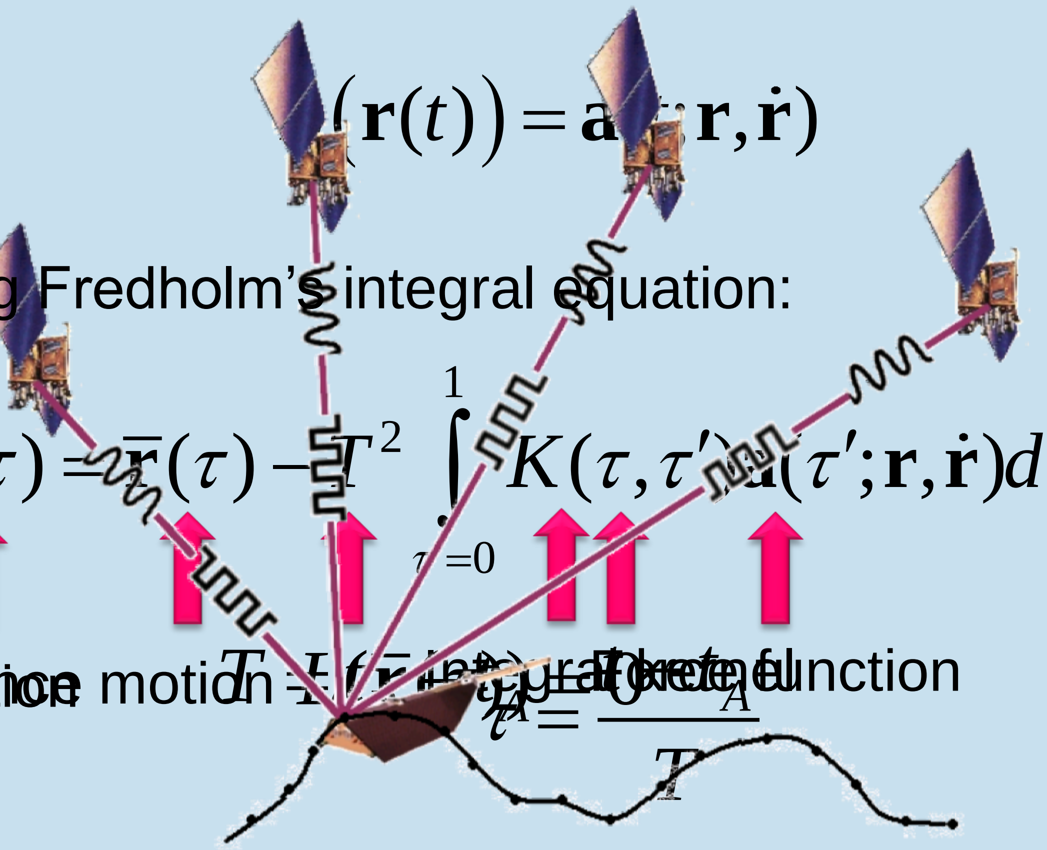


Equation of motion w.r.t self-adjoint differential operator:

$$\mathcal{L}\left(\mathbf{r}(t)\right) = \mathbf{a}(t;\mathbf{r},\dot{\mathbf{r}})$$

corresponding Fredholm's integral equation:

$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) - T^2 \int_{\tau'=0}^{1} K(\tau,\tau')\,\mathbf{a}(\tau';\mathbf{r},\dot{\mathbf{r}})\,d\tau'$$



Position

Reference motion

Integral kernel

Force function

$T = t_E - t_A$

$\tau = \frac{t - t_A}{T}$

A satellite short arc:

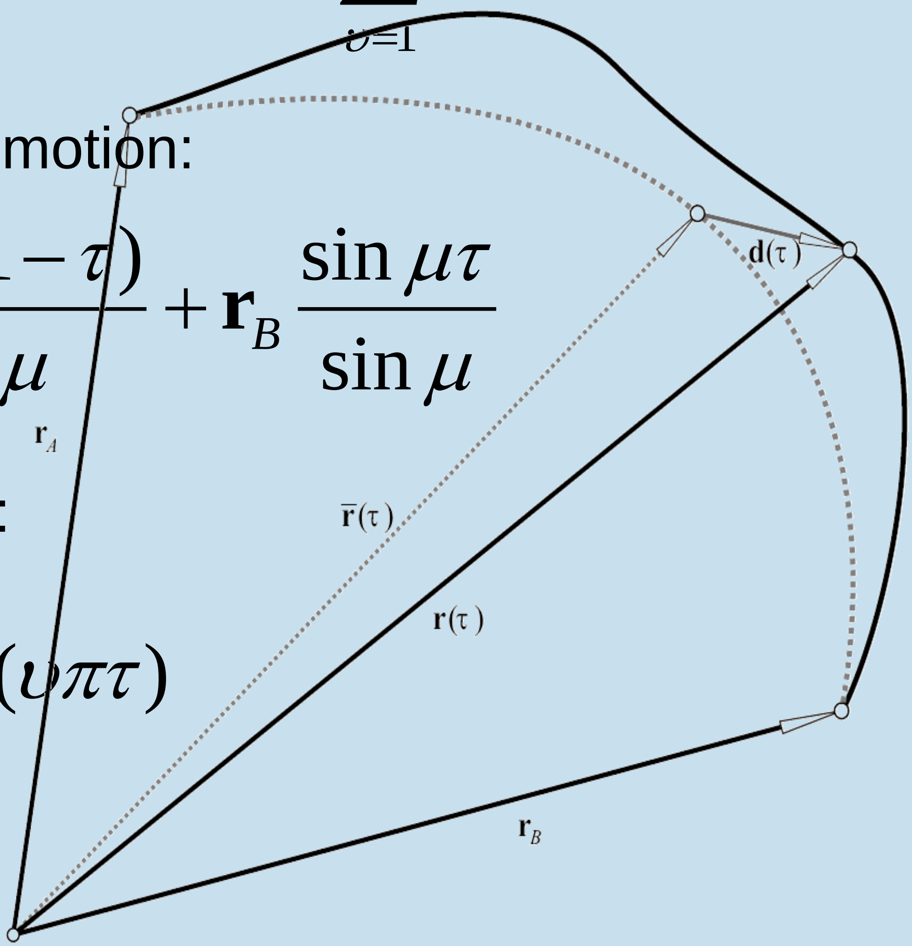
$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \mathbf{d}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{\upsilon=1}^{\infty} \mathbf{d}_\upsilon \sin(\upsilon\pi\tau)$$

elliptical reference motion:

$$\bar{\mathbf{r}}(\tau) = \mathbf{r}_A \frac{\sin\mu(1-\tau)}{\sin\mu} + \mathbf{r}_B \frac{\sin\mu\tau}{\sin\mu}$$

difference function:

$$\mathbf{d}(\tau) = \sum_{\upsilon=1}^{\infty} \mathbf{d}_\upsilon \sin(\upsilon\pi\tau)$$



Short arc representation – Fourier series

A satellite short arc can be represented:

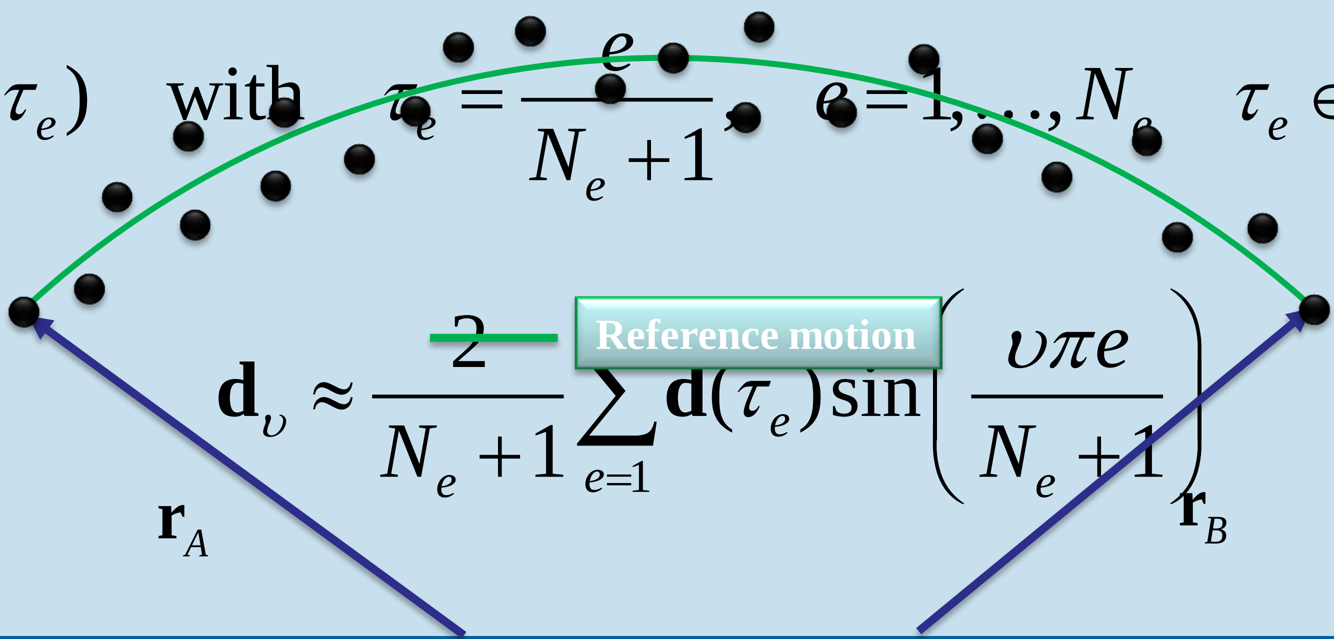
$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \mathbf{d}(\tau)$$

$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{\upsilon=1}^{\infty} \mathbf{d}_\upsilon \sin(\upsilon\pi\tau)$$

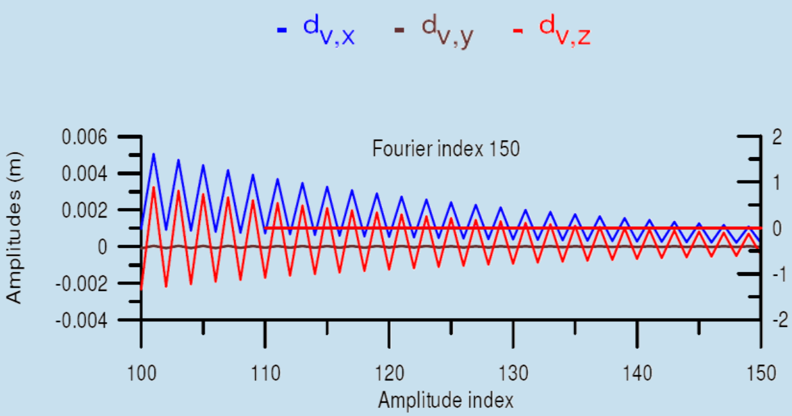
$$\mathbf{d}_\upsilon = 2 \int_{\tau'=0}^{1} \mathbf{d}(\tau) \sin(\upsilon\pi\tau')\, d\tau'$$

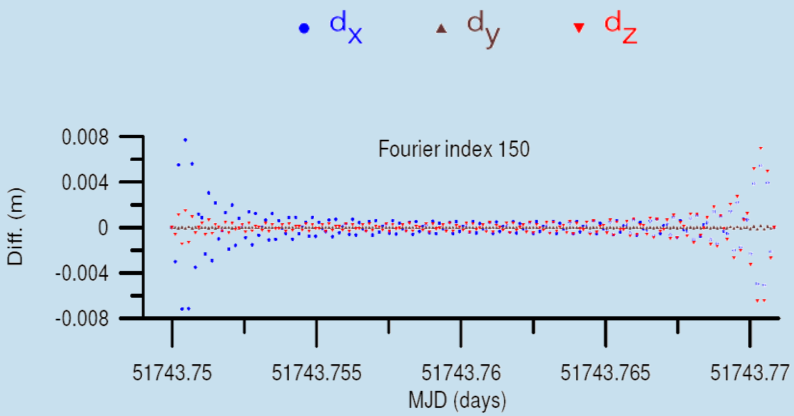
$$\mathbf{d}(\tau_e) \quad \text{with} \quad \tau_e = \frac{e}{N_e+1}, \quad e = 1, \ldots, N_e \quad \tau_e \in]0,1[$$

$$\mathbf{d}_\upsilon \approx \frac{2}{N_e+1} \sum_{e=1} \mathbf{d}(\tau_e) \sin\left(\frac{\upsilon\pi e}{N_e+1}\right)$$



Fourier analysis – ellipse mode





Amplitudes

Remainders

A satellite short arc can be represented:

$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{\upsilon=1}^{\infty} \mathbf{d}_{\upsilon} \sin(\upsilon\pi\tau)$$

with Fourier amplitudes:

$$\mathbf{d}_{\upsilon} = 2 \int_{\tau'=0}^{1} \mathbf{d}(\tau) \sin(\upsilon\pi\tau') d\tau'$$

Fourier series amplitudes:

$$\mathbf{d}_{\upsilon} = \sum_{j=1}^{J} \frac{2(-1)^{j+1}}{(\upsilon\pi)^{2j+1}} [(-1)^{\upsilon} \mathbf{d}^{[2j]}(1) - \mathbf{d}^{[2j]}(0)] +$$

$$+\beta \frac{2}{(\upsilon\pi)^{2J+1}} \int_{\tau'=0}^{1} \mathbf{d}^{[2J+2]}(\tau') \sin(\upsilon\pi\tau') d\tau'$$

Short arc representation – Euler-Bernoulli

$$\mathbf{d}_F^{\infty} = \mathbf{d}(\tau) = \sum_{\upsilon=1}^{\infty} \mathbf{d}_{\upsilon} \sin(\upsilon\pi\tau) =$$

$$= \sum_{j=1}^{\infty} \mathbf{e}_{2j} E_{2j}(\tau) + \sum_{j=1}^{\infty} \mathbf{b}_{2j+1} B_{2j+1}(\tau) = \mathbf{d}_P^{\infty}$$

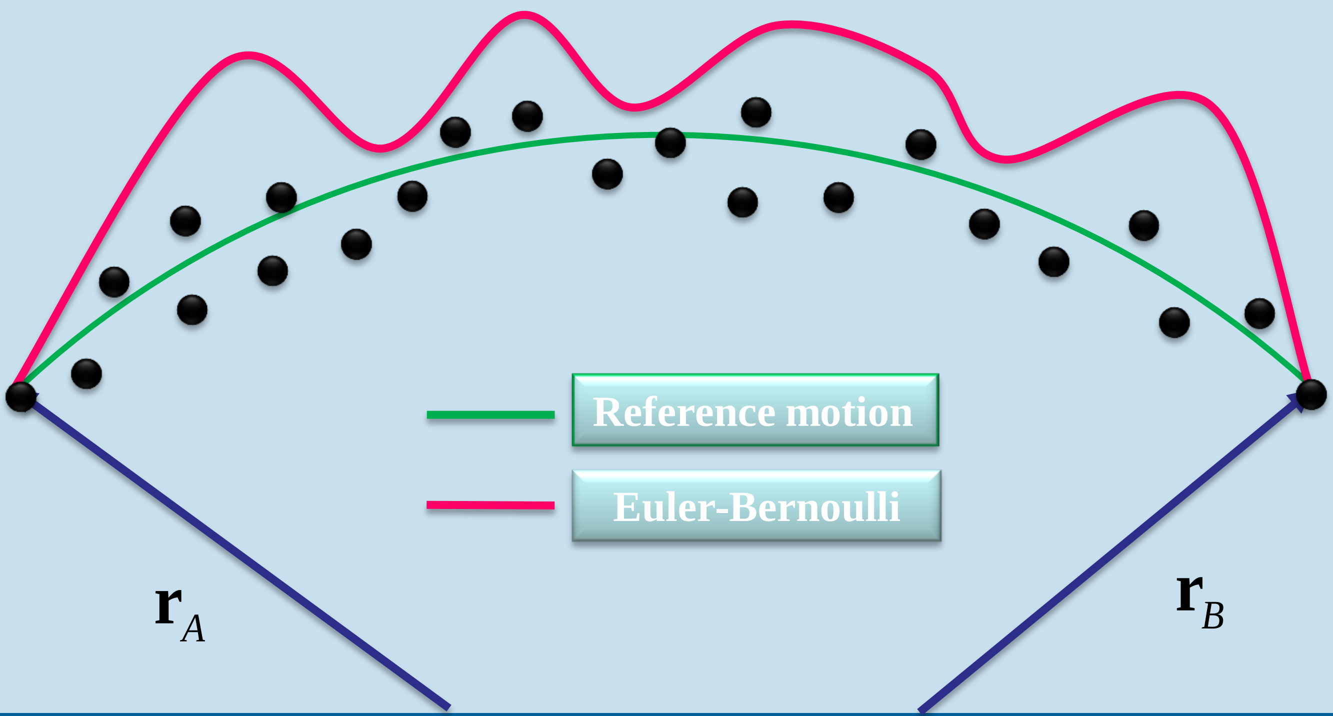
A satellite short arc can be represented:

$$\sum_{\upsilon=1}^{\infty} \mathbf{d}_{\upsilon} \sin(\upsilon\pi\tau) = \sum_{j=1}^{\infty} \mathbf{e}_{2j} E_{2j}(\tau) + \sum_{j=1}^{\infty} \mathbf{b}_{2j+1} B_{2j+1}(\tau)$$

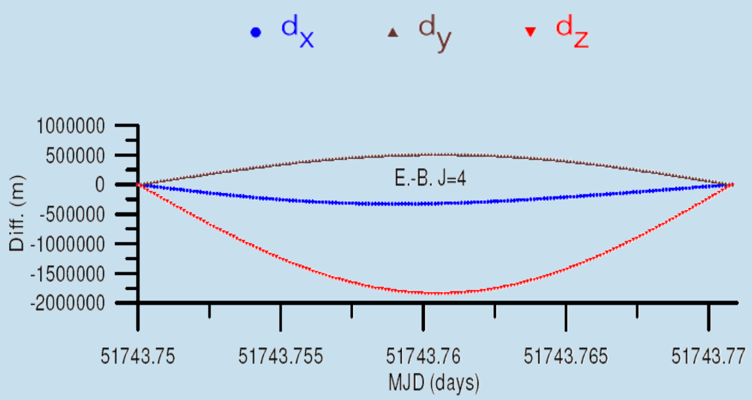
Determination of Euler-Bernoulli coefficients

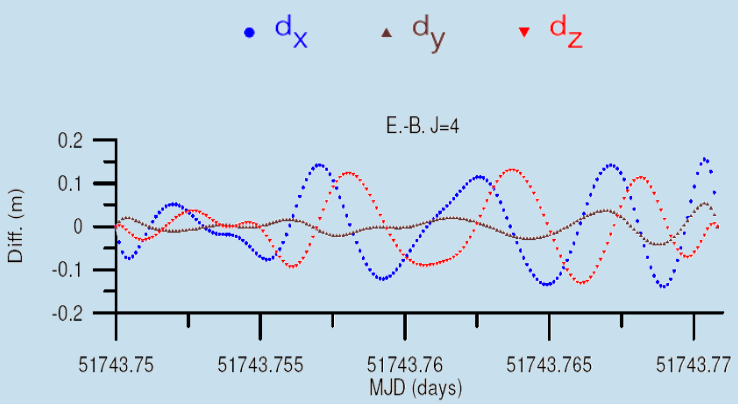
A satellite short arc can be represented with the Euler-Bernoulli term up to degree *J* as:

$$\mathbf{r}(\tau) - \bar{\mathbf{r}}(\tau) = \mathbf{d}(\tau) \approx \sum_{j=1}^{J} \mathbf{e}_{2j} E_{2j}(\tau) + \sum_{j=1}^{J} \mathbf{b}_{2j+1} B_{2j+1}(\tau)$$



Determination of Euler-Bernoulli coefficients





E.-B terms

Remainders

Short arc representation

LEO orbit can be represented as:

$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \mathbf{d}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{\upsilon=1}^{n} \mathbf{d}_\upsilon \sin(\upsilon\pi\tau)$$

Gibbs effect!

or

Precision!

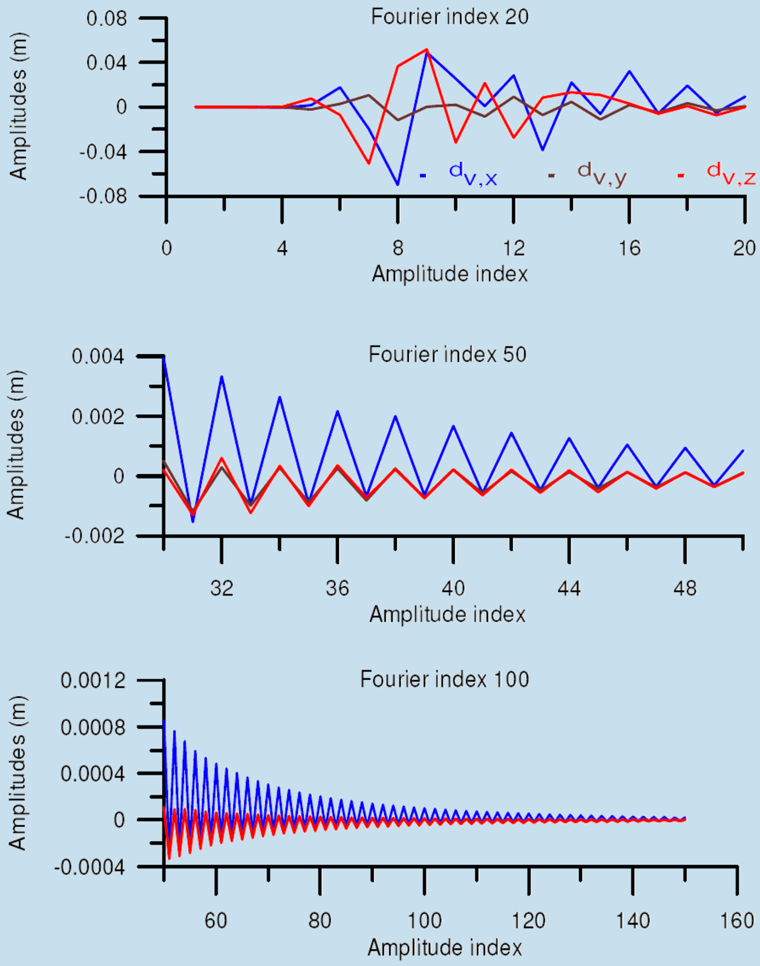
$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \mathbf{d}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{j=1}^{J} \mathbf{e}_{2j} E_{2j}(\tau) + \sum_{j=1}^{J} \mathbf{b}_{2j+1} B_{2j+1}(\tau)$$

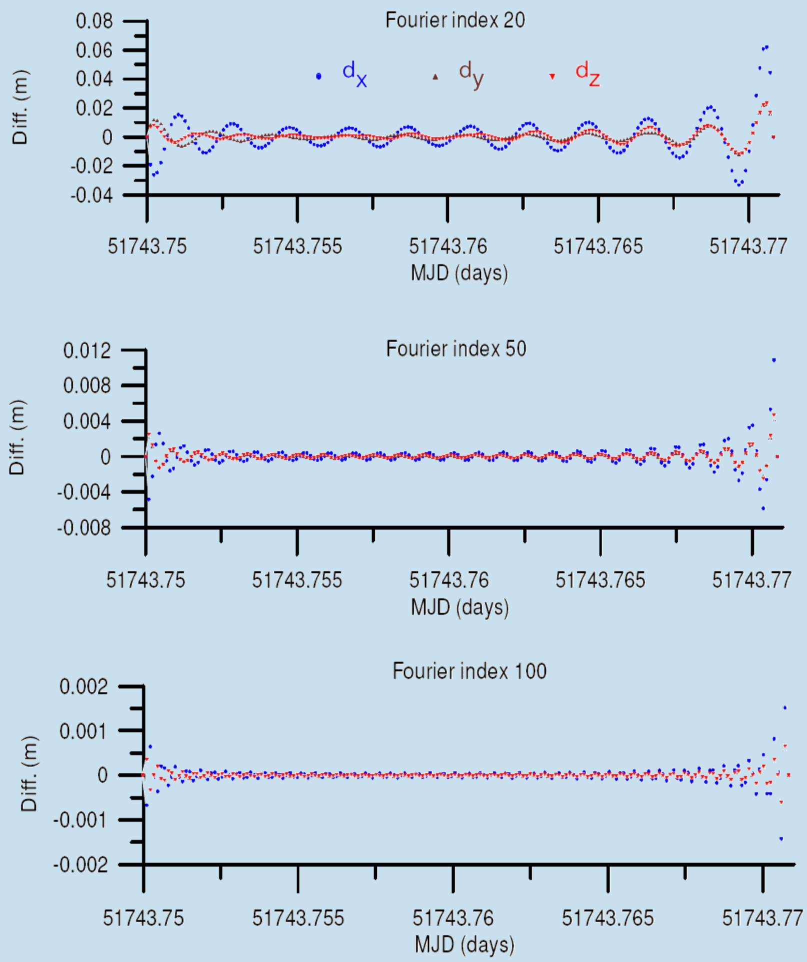
fast convergence!

Solution?

$$\mathbf{r}(\tau) = \bar{\mathbf{r}}(\tau) + \sum_{j=1}^{J} \mathbf{e}_{2j} E_{2j}(\tau) + \sum_{j=1}^{J} \mathbf{b}_{2j+1} B_{2j+1}(\tau) + \sum_{\upsilon=1}^{\bar{n}} \bar{\mathbf{d}}_\upsilon \sin(\upsilon\pi\tau)$$

Kinematical POD – ellipse mode, J=4





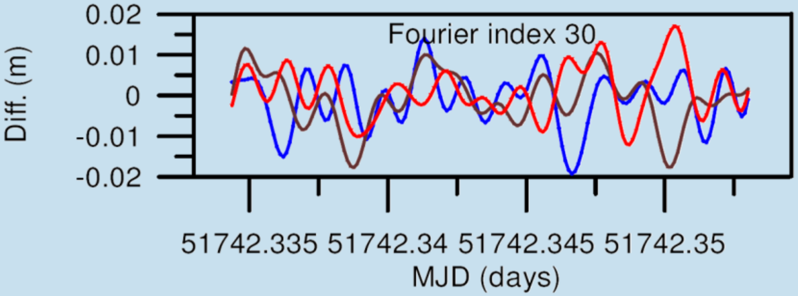
Amplitudes

Remainders

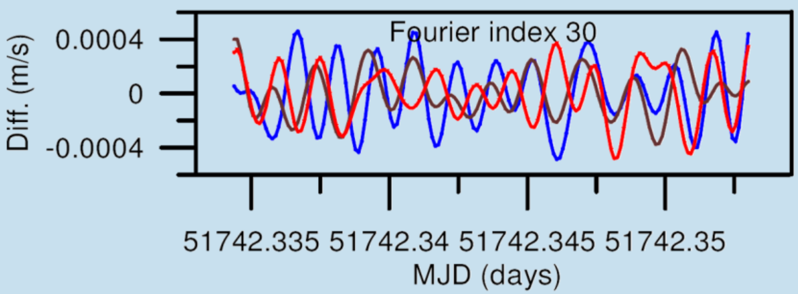
Processing procedures

- GPOD
- KPOD
  - Method
  - Simulation
  - Real data
- RKPOD
  - Method
  - Simulation
  - Real data
- DPOD

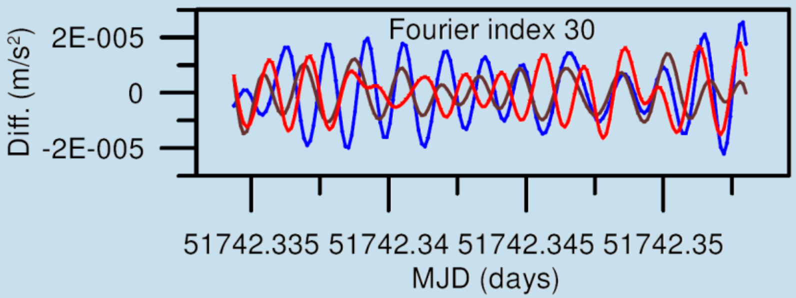
Kinematical POD – simulation (2cm noise SST)



Position differences



Velocity differences



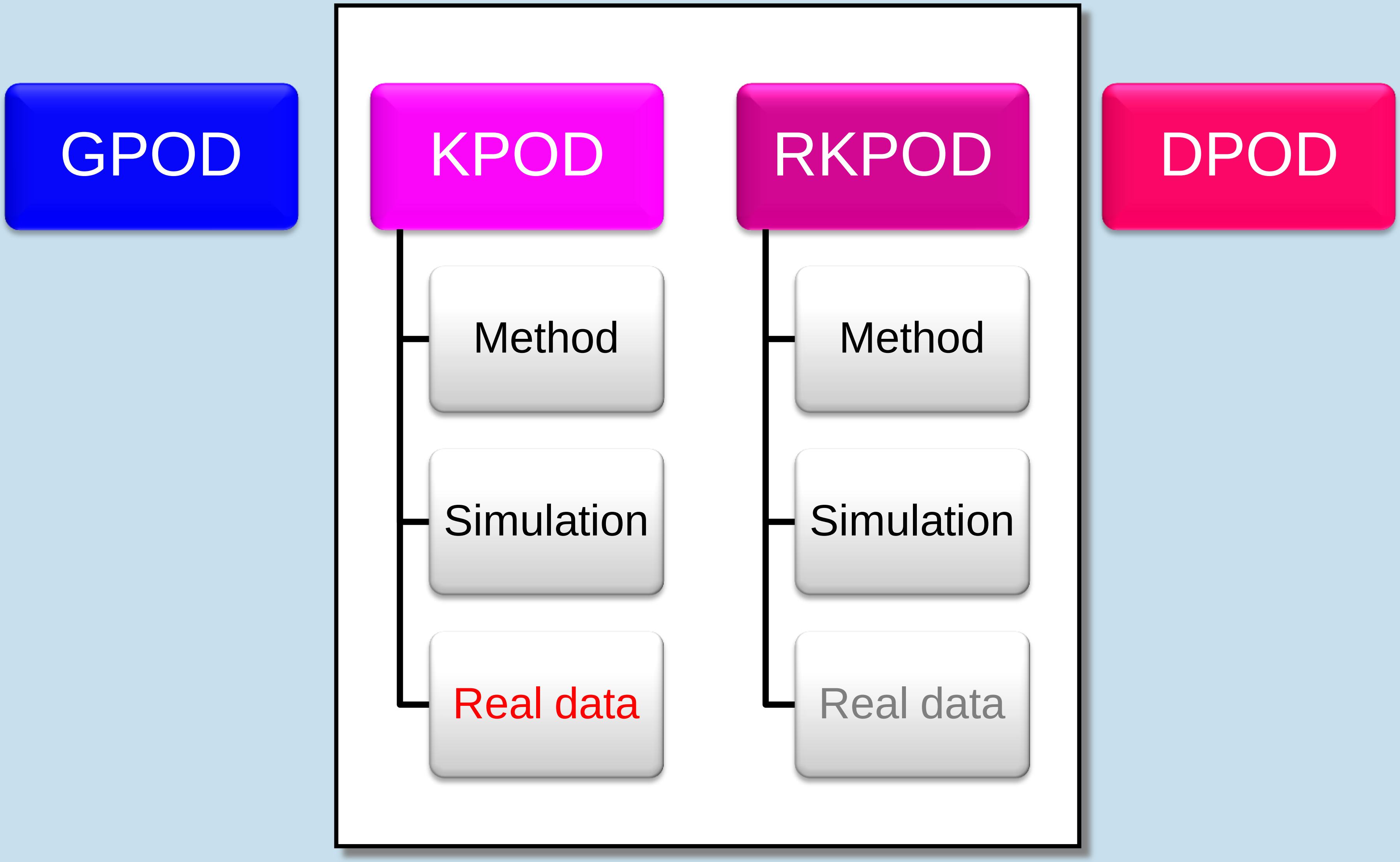
Acceleration differences

RMS

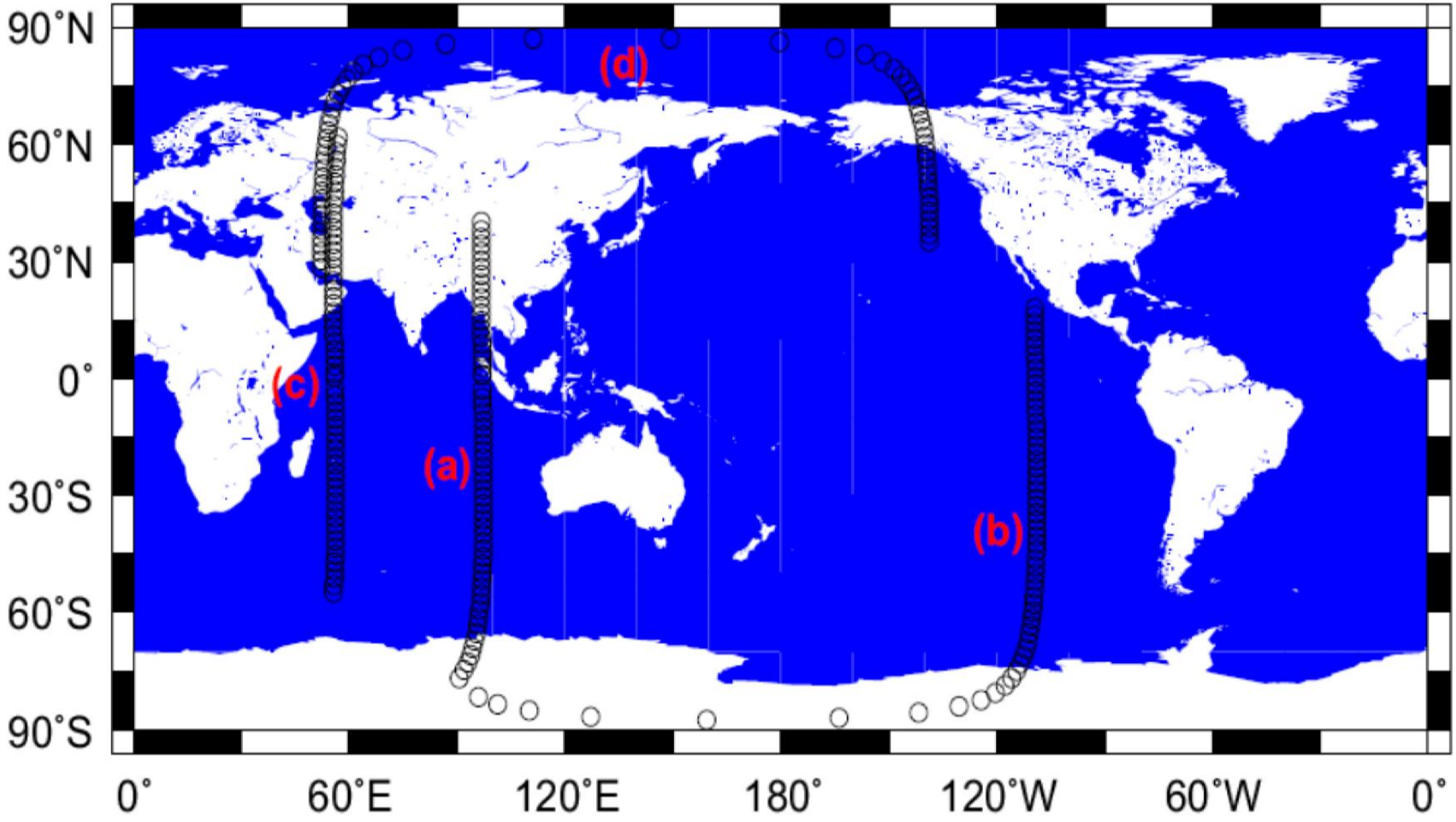
index	Pos.(m)	Vel.(m/s)	Acc.(m/s^2)
20	0.012644	0.000353	0.000012
30	0.010717	0.000397	0.000018
40	0.011997	0.000463	0.000025
59	0.014737	0.000941	0.000077

Statistical values

Processing procedures

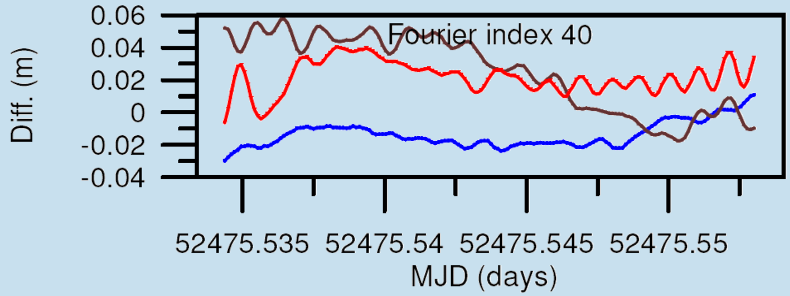


KPOD – real case

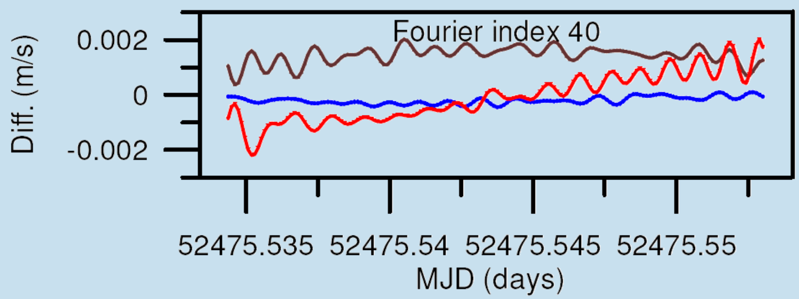


Four short arcs (30 min.) ground track of CHAMP

Kinematical POD – short arc case (b)



IGG - GFZ positions

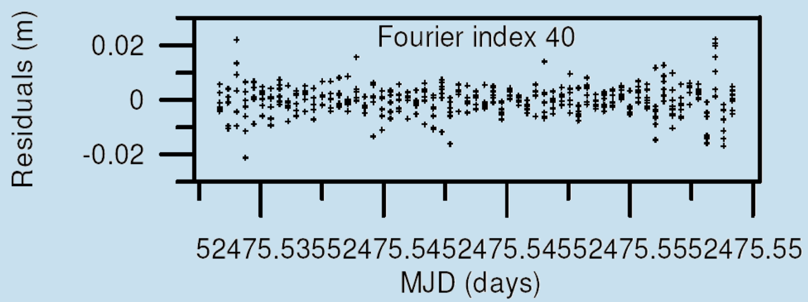


IGG - GFZ velocities

RMS

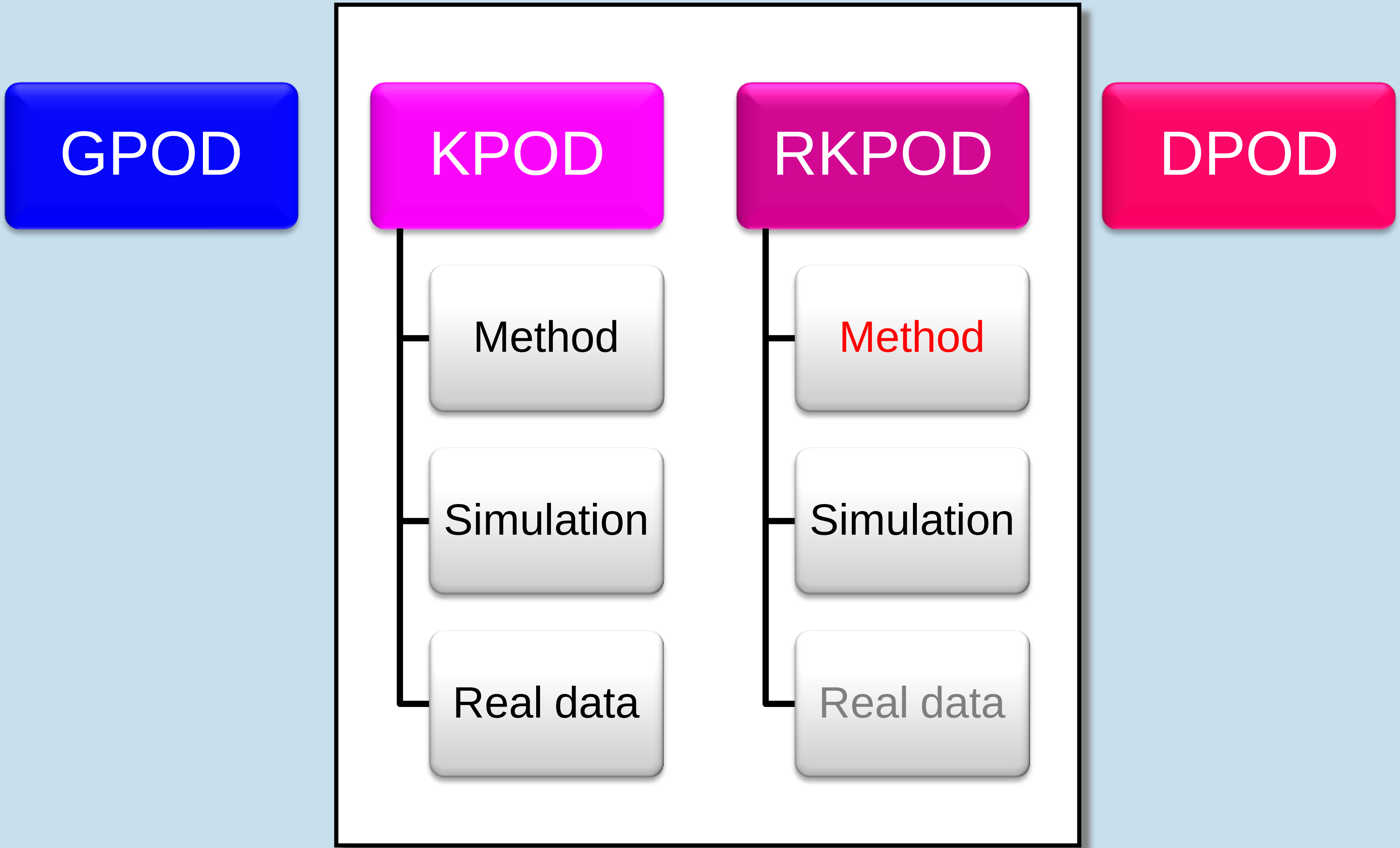
index	Pos. (m)	Vel. (m/s)
20	0.0503	0.0019
30	0.0455	0.0018
40	0.0449	0.0017
59	0.0449	0.0017

Statistical values



GPS-SST residuals

Processing procedures



Reduced-Kinematical POD – method

Dynamical info.

$$= \bar{\mathbf{r}}(\tau) + \sum_{\upsilon=1}^{n} \mathbf{d}_{\upsilon} \sin(\upsilon\pi\tau)$$

$$\tilde{\mathbf{d}}_{\upsilon} = -\frac{2T^{2}}{\upsilon^{2}\pi^{2} - \mu^{2}} \int_{\tau'=0}^{1} \sin(\upsilon\pi\tau)\mathbf{a}(\tau';\mathbf{r},\dot{\mathbf{r}})d\tau'$$

✓Introduction of an approximate force function $\left(\tilde{\mathbf{d}}_{i}\cdots\tilde{\mathbf{d}}_{j}\right)$, $\mathbf{C}_{\left(\tilde{\mathbf{d}}_{i}\cdots\tilde{\mathbf{d}}_{j}\right)}$

✓Fixing only some orbit parameters $\left(\tilde{\mathbf{d}}_{i}\cdots\tilde{\mathbf{d}}_{j}\right)$, $\mathbf{C}_{\left(\tilde{\mathbf{d}}_{i}\cdots\tilde{\mathbf{d}}_{j}\right)} \rightarrow \mathbf{0}$

✓Down- or up weighting $\mathbf{C}_{\left(\tilde{\mathbf{d}}_{1}\cdots\tilde{\mathbf{d}}_{n}\right)}$ in relation to $\mathbf{C}_{\left(\mathbf{d}_{1}\cdots\mathbf{d}_{n}\right)}$

Processing procedures

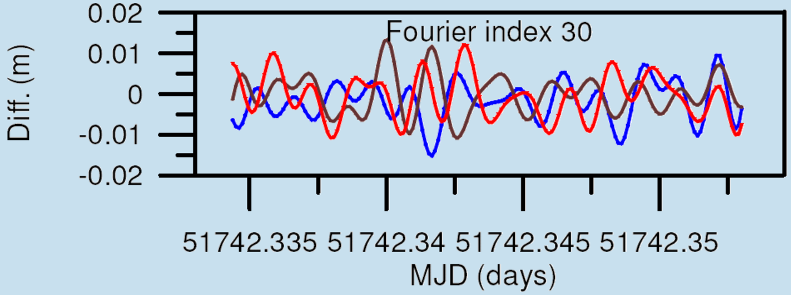
GPOD

KPOD
- Method
- Simulation
- Real data

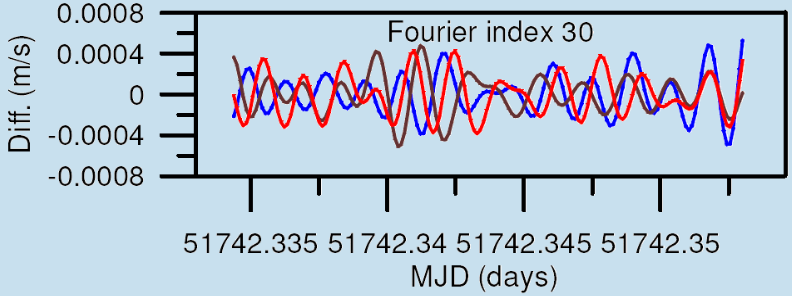
RKPOD
- Method
- Simulation
- Real data

DPOD

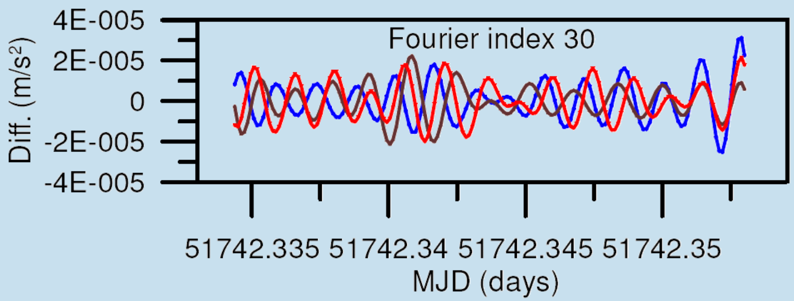
RKPOD – simulation (Amp. 1-5, 2cm, N=300)



Position differences



Velocity differences



Acceleration differences

RMS

index	Pos.(m)	Vel.(m/s)	Acc.(m/s^2)
20	0.012831	0.000316	0.000012
30	0.008873	0.000337	0.000016
40	0.014034	0.000402	0.000021
59	0.011553	0.000721	0.000056

Statistical values

Conclusions and recommendations

- GNSS-LEO satellites configuration and geometrical strength play an important role in POD,

- Kinematical POD can be used to recover the Earth's gravity field model based on the POD methods,

- No gravity field and no force models have been used in the Geometrical and Kinematical modes (advantage),

- The proposed kinematical orbit determination method is very flexible. A smooth transition from kinematical to reduced kinematical and finally dynamical or vice-versa is possible.

Thank you
for your attention