Classical and Quantum Gravity

NOTE • **OPEN ACCESS**

Gravitational wave hints black hole remnants as dark matter

To cite this article: Guillem Domènech and Misao Sasaki 2023 *Class. Quantum Grav.* **40** 177001

OPEN ACCESS
IOP Publishing
Classical and Quantum Gravity
Class. Quantum Grav. 40 (2023) 177001 (13pp)
https://doi.org/10.1088/1361-6382/ace493

Note

Gravitational wave hints black hole remnants as dark matter

Guillem Domènech[1,*] and **Misao Sasaki**[2,3,4]

[1] Institute for Theoretical Physics, Leibniz University Hannover, Appelstraße 2, 30167 Hannover, Germany
[2] Kavli Institute for the Physics and Mathematics of the Universe (WPI), The University of Tokyo Institutes for Advanced Study, The University of Tokyo, Chiba 277-8583, Japan
[3] Center for Gravitational Physics, Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan
[4] Leung Center for Cosmology and Particle Astrophysics, National Taiwan University, Taipei 10617, Taiwan

E-mail: guillem.domenech@itp.uni-hannover.de

Received 21 March 2023; revised 7 June 2023
Accepted for publication 5 July 2023
Published 26 July 2023

Abstract
The end state of Hawking evaporation of a black hole is uncertain. Some candidate quantum gravity theories, such as loop quantum gravity and asymptotic safe gravity, hint towards Planck sized remnants. If so, the Universe might be filled with remnants of tiny primordial black holes, which formed with mass $M < 10^9$ g. A unique scenario is the case of $M \sim 5\times 10^5$ g, where tiny primordial black holes reheat the Universe by Hawking evaporation and their remnants dominate the dark matter (DM). Here, we point out that this scenario leads to a cosmological gravitational wave signal at frequencies $\sim$100 Hz. Finding such a particular gravitational wave signature with, e.g. the Einstein telescope, would suggest black hole remnants as DM.

Keywords: primordial black holes, black hole remnants, gravitational waves, dark matter

(Some figures may appear in colour only in the online journal)

* Author to whom any correspondence should be addressed.

1. Introduction

In 1987, about a decade and a half years after Hawking and Carr [1, 2] showed that black holes could form in the early Universe, MacGibbon–in a paper published in Nature [3]–already entertained the possibility that cold dark matter (DM) could be made of residues left after black hole evaporation [4, 5]. The idea that black holes might not evaporate completely already appears by Man'ko and Markov in the proceedings of a singular workshop in Moscow in 1981, called 'Seminar on Quantum Gravity' [6] (the proceedings is worth checking). From today's perspective, the possibility of black hole remnants, sometimes also called relics, might not be the most attractive end state of a black hole, albeit it is a plausible one. After all, even if Hawking evaporation leaves something behind, it is not clear how one would directly detect Planck sized objects or prove their existence. Nevertheless, their collective gravitational pull could, e.g. explain a completely invisible DM [3] and their bag-of-gold interior might offer a solution to the black hole information loss paradox [7]; the latter is still under debate [8]. We refer the reader to [9] for a thorough review on the information loss paradox and black hole remnants.

From a theoretical point of view, remnants might be a consequence of quantum gravity. There is some degree of belief that the theory of quantum gravity would be free from singularities, such as those that appear in the interior of a black hole. Within such candidate quantum gravity theories, a regular black hole might well end up in a stable state after evaporation (see e.g. [10]). For instance, regular black hole solutions have been found within loop quantum gravity [11–13], non-commutative geometry [14], limiting curvature models in mimetic gravity [15–17] and generalized uncertainty principles [18]. For more models, see the collection in [9, 19, 20]. Recently, there is also growing interest within asymptotic safe gravity (see [21, 22] for recent reviews). Note that these are static solutions and it is not clear what would occur when considering the initial collapse of matter.

From a cosmologist perspective, black hole remnants appear to be a 'bonus' to the rich phenomenology of primordial black holes (PBHs) for short (curious fact: the usage of the acronym PBH dates back to 1975 [23], but in lower case letters). The PBH scenario is nowadays a very popular topic, as can be seen from the many recent (and thorough) reviews [24–29]. We will be most concerned with tiny ($= M_{\rm PBH} < 5\times10^8$ g) PBHs, since they evaporate much before Big Bang Nucleosynthesis (BBN) [30–32] and may abundantly leave remnants. Though we will not dwell into the details of the formation of such tiny PBHs, they seem to be easily generated towards the end of inflation by preheating instabilities or quantum stochastic effects [33–35].

Tiny PBHs have an interesting early Universe cosmology: they could totally (or partially) reheat the Universe [36–39] and explain the baryon asymmetry of the Universe [40–52]. Tiny PBHs also produce high frequency gravitational waves (GWs) by Hawking evaporation [45, 53–61], PBH binaries [54, 57] and secondary GWs [57, 62–68]. See [57, 65] for a recollection of GWs associated with tiny PBHs. For the original works see Carr [36], Chapline [37], Zeldovich and Starobinsky [42, 43] and Turner [40, 41]. On top of all that, PBH remnants[5] could account for a fraction or all of the DM, if they exist. Any overproduction of remnants would then constrain inflationary models, e.g. see [73, 74].

Very interestingly, it has been recently noticed that some GW products of tiny PBHs might be within the range of future experiments, such as CMB-S4 [56, 58–61] and GW detectors such as the Einstein Telescope [57, 62–68] and, perhaps, high frequency GW detectors [49, 61]. In the future, we may find signatures of PBH evaporation in the early Universe. In this note, we

[5] For a brief recent discussion on how PBHs remnants would not recoil from Hawking evaporation see [69–71]. For PBH formation in loop quantum gravity see [72].

add a unique signature to the PBH remnant scenario. We show that density fluctuations due to the initial inhomogeneous distribution of PBHs leads to (induced) GWs within a fixed low frequency range, which enters the LIGO/Virgo band and could be detected in the future by ET. The rest of the note is organized as follows. We briefly review the PBH remnant scenario as DM and the production of low frequency GWs in section 2. We then end with a short discussion in section 3. Most of the details of the formulas in this paper can be found, e.g. in [25, 47, 57, 65, 73]. When needed we use the cosmological parameters of Planck 2018 [75].

2. PBH remnants and low frequency GWs

In the tiny PBH scenario we have two basic parameters: the initial mass of the PBHs at formation $M_{\rm PBH,f}$ and the initial energy density fraction $\beta = \rho_{\rm PBH,f}/\rho_{\rm total}$ [25]. For simplicity, we will assume that PBHs form by the collapse of primordial fluctuations with a monochromatic PBH mass function. We discuss later the effects of extended mass functions. Under this assumption, $M_{\rm PBH,f}$ and β are related to the Hubble horizon H and the number density of PBHs $n_{\rm PBH}$ at formation[6]. For a fixed $M_{\rm PBH,f}$ and β we have

$$H_{\rm f} = 4\pi\gamma \frac{M_{\rm pl}^2}{M_{\rm PBH,f}} \approx 5\times 10^{13}\,{\rm GeV} \times \left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^{-1}, \tag{1}$$

where we used that $\gamma \sim 0.2$ [25] and that the reduced Planck mass is given by $M_{\rm pl} = (8\pi G)^{-1/2} \approx 4.235\times 10^{18}\,{\rm GeV} \approx 4.3\times 10^{-6}\,{\rm g}$. For high energy scale inflation one approximately has $H_{\rm f} \sim 10^{-5} M_{\rm pl}$ and so at least $M_{\rm PBH,f} > 1\,{\rm g}$. We also have that[7]

$$n_{\rm PBH,f} = \frac{\rho_{\rm PBH,f}}{M_{\rm PBH,f}} = \frac{3\beta}{4\pi\gamma} H_{\rm f}^3 \approx 10^{-3}{\rm qm}^{-3} \times \beta \left(\frac{1\,{\rm g}}{M_{\rm PBH,f}}\right)^3, \tag{2}$$

where we used $\rho_{\rm total,f} = 3H_{\rm f}^2 M_{\rm pl}^2$ and equation (1).

After formation, tiny PBHs quickly evaporate. The evaporation time reads [57]

$$t_{\rm eva} \approx \frac{160\alpha}{3.8\pi\, g_H(T_{\rm PBH})} \frac{M_{\rm PBH,f}^3}{M_{\rm pl}^4} \approx 400\,{\rm qs} \times \alpha \left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^3, \tag{3}$$

where $g_H(T_{\rm PBH})$ is the spin-weighted degrees of freedom and we introduced the parameter α to take into account the effect of the PBH spin. For no spin $\alpha = 1$, while for a near extremal PBH $\alpha \sim 1/2$ [76]. In deriving equation (3) we assumed that the evaporation time is much larger than the formation time and that $g_H(T_{\rm PBH}) \approx 108$ [56]. If we compare the evaporation time in equation (3) with the Hubble time at formation, equation (1), in a radiation-dominated Universe,

$$t_{\rm f} \approx \frac{1}{2H_{\rm f}} \approx 10^{-8}\,{\rm qs} \times \left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right), \tag{4}$$

we indeed see that $t_{\rm eva} \gg t_{\rm f}$.

[6] It is also useful to write $M_{\rm PBH,f}$ in grams in terms of $H_{\rm f}$ which gives

$$M_{\rm PBH,f} \approx 10^{-5}\,{\rm g} \times \frac{M_{\rm pl}}{H_{\rm f}}.$$

[7] Since 2022 the International Bureau of Weights and Measures (BIPM) introduced new prefixes for the metric system. There 'qm' stands for quectometer defined by ${\rm qm} = 10^{-30}\,{\rm m}$, about 10^5 times larger than the Planck length. Similarly, 'qs' for quectosecond, ${\rm qs} = 10^{-30}\,{\rm s}$.

Many of the discussions that follow will depend on the ratio $t_{\rm eva}/t_f$, which tells us how long these tiny PBHs stayed around. So, let us write it explicitly:

$$R_{\rm eva,\,f} \equiv \frac{t_{\rm eva}}{t_{\rm f}} = \frac{1280\pi\gamma\alpha}{3.8\pi\, g_H(T_{\rm PBH})}\frac{M^2_{\rm PBH,f}}{M^2_{\rm pl}} \approx 3\times 10^{10}\,\alpha\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^2 . \tag{5}$$

For instance, the condition for the PBH dominance is given by

$$\beta > \beta_{\rm min} = 1/\sqrt{R_{\rm eva,\,f}} \approx 6\times 10^{-6}\alpha^{-1/2}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^{-1} . \tag{6}$$

If $\beta < \beta_{\rm min}$, PBHs never dominate. The minimum abundance $\beta_{\rm min}$ is obtained by requiring that $H_{\rm eeq} > H_{\rm eva}$, where $H_{\rm eeq}$ and $H_{\rm eva}$ respectively refer to the Hubble parameter at the time of early radiation-PBH equality after PBH formation and the Hubble parameter right after the complete PBH evaporation. We then used that $H_{\rm eeq}/H_{\rm f} \approx \sqrt{2}\beta^2$ and $H_{\rm eva} \sim 1/t_{\rm eva}$. The $\sqrt{2}$ in $H_{\rm eeq}$ appears after using the exact solutions for a radiation-matter Universe (see, e.g. equation (1.81) in Mukhanov's book [77]) which is a good approximation since PBHs evaporate almost instantaneously [57]. We can also find the amount of the expansion of the Universe from PBH formation until PBH evaporation,

$$\frac{a_{\rm eva}}{a_{\rm f}} \approx \begin{cases} \sqrt{R_{\rm eva,f}} \approx 2\times 10^5\,\alpha^{1/2}\left(\dfrac{M_{\rm PBH,f}}{1\,{\rm g}}\right); & \beta < \beta_{\rm min}\,, \\ \left(\dfrac{3\beta^{1/2}R_{\rm eva,f}}{4}\right)^{2/3} \approx 9\times 10^6\,\beta^{1/3}\alpha^{2/3}\left(\dfrac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^{4/3}; & \beta > \beta_{\rm min}\,, \end{cases} \tag{7}$$

where, for the case $\beta > \beta_{\rm min}$, we used the fact that $H = 2/(3t)$ in a matter dominated Universe. For a radiation dominated Universe, one instead has $H = 1/(2t)$. We see that larger PBH masses lead to larger number of e-folds. The β dependence in the case $\beta > \beta_{\rm min}$ can be understood from the fact that the early radiation-PBH equality depends on β.

We can also compute the temperature of the radiation filling the Universe after PBH evaporation. This is given by

$$T_{\rm eva} \approx 2.4(2.8)\times 10^{10}\,{\rm GeV}\,\alpha^{-1/2}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^{-3/2}\left(\frac{g_{s\star}(T_{\rm eva})}{106.75}\right)^{-1/4} , \tag{8}$$

where the coefficient 2.4(2.8) is for the case $\beta < \beta_{\rm min}(\beta > \beta_{\rm min})$, $g_{s\star}$ is the effective degrees of freedom for the entropy and we used the Planck 2018 [75] value for $H_{\rm eq}$. Later $g_\star$ denotes the effective degrees of freedom for the energy density. To evaluate them we use the fitting formulas provided in [78]. Evaporation before BBN, that is $T_{\rm eva} > 4\,{\rm MeV}$ [30–32], requires $M < 5\times 10^8$ g. That is all we need to understand the PBH remnant scenario and the associated low frequency GWs.

2.1. GWs after PBH domination and evaporation

PBH formation is rather a rare event. Only those Hubble patches with high enough density contrast will collapse; for Gaussian fluctuations this is exponentially unlikely. Thus, the spatial distribution of PBH formation is to a good approximation uniformly random. This leads to a Poisson type spectrum of PBH number density isocurvature fluctuations [62] (see figure 1 for

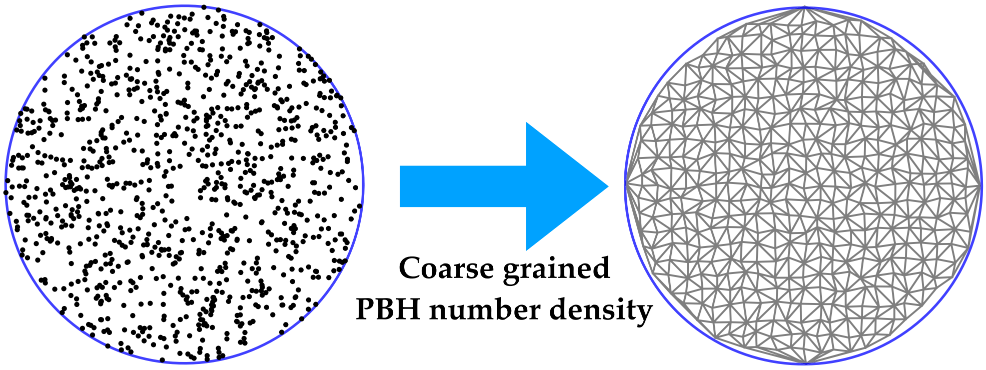

Figure 1. PBHs form randomly in the early Universe approximately according to a uniform distribution. In a coarse grained fluid picture this leads to PBH number density fluctuations. These fluctuations are isocurvature in nature because PBH formation leave holes in the original radiation fluid. These fluctuations are responsible for a large production of induced GWs.

an illustration). Note, however, that the coarse grained, fluid picture for the collection of PBHs breaks down at the mean inter-PBH comoving separation, which is given by

$$k_{\rm uv} \equiv a_{\rm f}\bar{d}_{\rm f}^{-1} = \left(\frac{4\pi}{3}n_{\rm PBH,\,f}a_{\rm f}^3\right)^{1/3} = \frac{\beta^{1/3}}{\gamma^{1/3}}a_{\rm f}H_{\rm f}\,. \tag{9}$$

Equation (9) gives us the high momenta cut-off for the spectrum of number density fluctuations. This is also where fluctuations are larger and the spectrum of (PBH-isocurvature) induced GWs peaks[8]. We consider from now on, in this subsection, only the case where PBHs dominate the Universe, i.e. $\beta > \beta_{\rm min}$. Otherwise, the production of induced GWs is very much suppressed and one needs very large isocurvature fluctuations [79].

Curiously, when PBHs dominate, there is a β independent relation between $k_{\rm uv}$ and $k_{\rm eva}$, which is given by [63]

$$\frac{k_{\rm uv}}{k_{\rm eva}} = \frac{\beta^{1/3}}{\gamma^{1/3}}\frac{H_{\rm f}}{H_{\rm eva}}\frac{a_{\rm f}}{a_{\rm eva}} = \left(\frac{3R_{\rm eva,f}}{4\gamma}\right)^{1/3} \approx 5000\,\alpha^{1/3}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^{2/3}. \tag{10}$$

This allows us to easily find the peak frequency of the GWs today by using that

$$f_{\rm eva} = \frac{k_{\rm eva}}{2\pi a_0} \approx 3\times10^{-5}\,{\rm Hz}\left(\frac{T_{\rm eva}}{1{\rm TeV}}\right)\left(\frac{g_\star(T_{\rm eva})}{106.75}\right)^{1/2}\left(\frac{g_{s\star}(T_{\rm eva})}{106.75}\right)^{-1/3}$$
$$\approx 734\,{\rm Hz}\,\alpha^{-1/2}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^{-3/2}\left(\frac{g_\star(T_{\rm eva})}{106.75}\right)^{1/4}\left(\frac{g_{s\star}(T_{\rm eva})}{106.75}\right)^{-1/3}\gamma\,, \tag{11}$$

[8] The formation of tiny PBHs from curvature fluctuations also generates induced GWs but those are very high frequency.

which yields

$$f_{\rm uv} \approx 3.6\times 10^6\,{\rm Hz}\,\alpha^{-1/6}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^{-5/6}\left(\frac{g_\star(T_{\rm eva})}{106.75}\right)^{1/4}\left(\frac{g_{s\star}(T_{\rm eva})}{106.75}\right)^{-1/3}. \quad (12)$$

The calculation on the dominant contribution to the amplitude of GWs can be found in [63] (see also [62, 80, 81]). Here we only sketch the production of induced GWs and their amplitude after a PBH dominated Universe. Let us first give the result and then explain it. The induced GW spectrum from a PBH dominated Universe evaluated today is given by

$$\Omega_{\rm GW,0}h^2 = 1.62\times 10^{-5}\,c_g\,\Omega^{\rm peak}_{\rm GW,eva}\left(\frac{k}{k_{\rm uv}}\right)^{11/3}\Theta(k-k_{\rm uv})\,, \quad (13)$$

where

$$c_g \equiv \left(\frac{\Omega_{\rm r,0}h^2}{4.18\times 10^{-5}}\right)\left(\frac{g_\star(T_{\rm eva})}{106.75}\right)\left(\frac{g_{s\star}(T_{\rm eva})}{106.75}\right)^{-4/3}, \quad (14)$$

which takes into account the redshift of the GW energy density from evaporation until today as well as the change of relativistic degrees of freedom, $\Omega_{\rm r,0}h^2$ is the energy fraction of radiation today from Planck 2018 [75], and

$$\Omega^{\rm peak}_{\rm GW,eva} \approx \frac{\beta^{16/3}\gamma^{8/3}}{1536\times 2^{1/3}\sqrt{3}\pi}\left(\frac{k_{\rm uv}}{k_{\rm eva}}\right)^{17/3} \approx \frac{\alpha^{17/9}}{10}\left(\frac{\beta}{10^{-3}}\right)^{16/3}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^{34/9}, \quad (15)$$

which gives the peak amplitude right after PBH evaporation.

Now, let us roughly explain the origin of the amplitude equation (15). First of all, the factor $\beta^{16/3}$ is the suppression due to the fact that PBHs are formed during the radiation era and the amplitude of curvature fluctuations decay until PBHs dominate the Universe. The suppression goes as $\sim(k_{\rm eeq}/k_{\rm uv})^2 \propto \beta^{4/3}$, where $k_{\rm eeq} = a_{\rm eeq}H_{\rm eeq}$ is the mode that enters the horizon at the early radiation-PBH equality. Induced GWs, as a secondary effect, are proportional to the four-point function of curvature fluctuations. Hence, we obtain $(\beta^{4/3})^4 = \beta^{16/3}$. The factor involving $k_{\rm uv}/k_{\rm eva}$ is more interesting. During PBH domination, the curvature perturbation is constant on all scales and density fluctuations grow proportional to the scale factor. On scales corresponding to $k_{\rm uv}$, the PBH number density fluctuations at evaporation have grown by a factor $(k_{\rm uv}/k_{\rm eva})^2$, which is very large. And then, PBHs almost suddenly evaporate: huge pressureless density fluctuations are converted into huge radiation fluctuations which generates a huge wake in the velocities of the radiation fluid. For the induced GW spectrum, which is proportional to four gradients of velocities, we get $((k_{\rm uv}/k_{\rm eva})^2)^4 = (k_{\rm uv}/k_{\rm eva})^8$. To go from 8 to 17/3 one needs to account that only a very narrow window of scalar modes contribute to the integral, that is only those modes close enough to the resonance in the GW kernel, which is proportional to $k_{\rm eva}/k_{\rm uv}$ [57, 64, 80], this gets us to 7. The rest comes from the 'almost' sudden evaporation [57]: very short wavelength modes actually feel the time dependence of Hawking evaporation, which goes as $M_{\rm PBH}(t)\sim(1-t/t_{\rm eva})^{1/3}$, and suppresses curvature fluctuations by a factor $(k_{\rm eva}/k_{\rm uv})^{1/3}$. Thus, we have $8-1-4\times 1/3 = 17/3$. We can then use equation (15) to place upper bounds on β from BBN constraints, namely [63]

$$\beta < 10^{-3}\left(\frac{M_{\rm PBH,f}}{1\,{\rm g}}\right)^{-17/24}. \quad (16)$$

It is important to emphasize that equation (15) should be understood as a rough order of magnitude estimate, because at some point during PBH domination, the number density fluctuations exceed unity. Furthermore, the amplitude is very sensitive to the width of the PBH mass function. For a log-normal with logarithmic width $\sigma \sim 1$ the amplification is negligible [57]. In any case, let us note that for the parameters of interest, the curvature perturbation and its time derivative are always smaller than unity and well within the perturbative regime.

2.2. PBH remnants as DM

Let us collect all the previous results and assume that PBH evaporation leaves behind Planck relics with mass

$$m_{\rm relic} = rM_{\rm pl}\,, \tag{17}$$

where $r > 1$ is a free parameter. The fact that evaporation stops when $M_{\rm PBH}(t_{\rm end}) = m_{\rm relic}$ does not affect the calculations in section 2.1. That is, $t_{\rm end} = t_{\rm eva} - \delta t$, where $\delta t / t_{\rm eva} = O(t_{\rm eva} M_{\rm pl})$, which is negligibly tiny. Thus, let us use the previous results and require that PBH remnants occupy a fraction $f_{\rm relic}$ of the total DM today, that is

$$f_{\rm relic} \equiv \frac{\rho_{\rm relic}}{\rho_{\rm DM}}\,. \tag{18}$$

Extrapolating backwards from today until evaporation, using that $\rho_{\rm DM} \propto a^{-3}$, we have that

$$\begin{aligned}\Omega_{\rm relic}\big|_{\rm eva} &\approx 7\times 10^{-13} f_{\rm relic}\left(\frac{T_{\rm eva}}{1{\rm TeV}}\right)^{-1}\left(\frac{g_{s\star}(T_{\rm eva})}{106.75}\right)\left(\frac{g_{\star}(T_{\rm eva})}{106.75}\right)^{-1}\\ &\approx 2.9\,(2.5)\times 10^{-20} f_{\rm relic}\sqrt{\alpha}\left(\frac{M_{\rm PBH,f}}{1{\rm g}}\right)^{-3/2}\left(\frac{g_{s\star}(T_{\rm eva})}{106.75}\right)\left(\frac{g_{\star}(T_{\rm eva})}{106.75}\right)^{-3/4}\,,\end{aligned} \tag{19}$$

where again the value between brackets is for $(\beta > \beta_{\rm min})$ and we used that $\rho_{\rm DM,eq} = 3H_{\rm eq}^2 M_{\rm pl}^2/2$. If we now extrapolate forwards from the PBH formation, we have that at evaporation

$$\Omega_{\rm relic}\big|_{\rm eva} = \frac{\beta m_{\rm relic}}{M_{\rm PBH,f}}\frac{H_{\rm f}^2}{H_{\rm eva}^2}\frac{a_{\rm f}^3}{a_{\rm eva}^3} \approx \begin{cases}\dfrac{m_{\rm relic}}{M_{\rm PBH,f}}\beta\sqrt{R_{\rm eva,f}} \approx r\beta\sqrt{\alpha} & (\beta < \beta_{\rm min})\\ \dfrac{m_{\rm relic}}{M_{\rm PBH,f}} & (\beta > \beta_{\rm min})\end{cases}. \tag{20}$$

In this way, we can draw the parameter space where PBH remnants can be a fraction $f_{\rm relic}$ of DM. In general we find that for a fixed $f_{\rm relic}$,

$$M_{\rm PBH,f} \approx 10^6\,{\rm g}\begin{cases}2.4\dfrac{r^{2/3}}{f_{\rm relic}^{2/3}}\left(\dfrac{\beta}{10^{-10}}\right)^{2/3}\left(\dfrac{g_{s\star}(T_{\rm eva})}{106.75}\right)^{-2/3}\left(\dfrac{g_{\star}(T_{\rm eva})}{106.75}\right)^{1/2} & (\beta < \beta_{\rm min})\\ 0.5\dfrac{r^{2/5}}{f_{\rm relic}^{2/5}\alpha^{1/5}}\left(\dfrac{g_{s\star}(T_{\rm eva})}{106.75}\right)^{-2/5}\left(\dfrac{g_{\star}(T_{\rm eva})}{106.75}\right)^{3/10} & (\beta > \beta_{\rm min})\end{cases}. \tag{21}$$

We show the parameter space in figure 2.

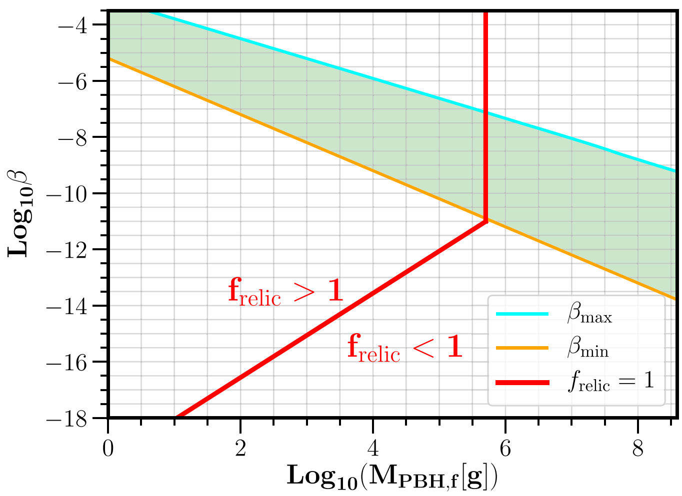

Figure 2. Parameter space in terms of the two basic parameters of the model, β and $M_{\mathrm{PBH,f}}$. The orange line shows the minimum value of β (6) to have PBH domination. The cyan line comes from requiring that induced GWs are not overproduced at BBN (16). The shaded region show the allowed parameter space to have PBH reheating. The red line shows the required values so that PBH remnants are the total dark matter (21).

Note that, if there was a PBH dominated stage in the early Universe ($\beta > \beta_{\mathrm{min}}$), and if the PBH remnants totally account for the DM ($f_{\mathrm{relic}} = 1$), the initial PBH mass is uniquely determined as

$$M_{\mathrm{PBH,f}} \approx 5\times 10^5\,\mathrm{g}\times \frac{r^{2/5}}{\alpha^{1/5}}\,. \tag{22}$$

This correspond to an evaporation temperature of $T_{\mathrm{eva}} \approx 80\,\mathrm{GeV}$. Note that this value is in agreement with [47]. This case corresponds to an induced GW signal with peak at

$$f_{\mathrm{uv}} \approx 70\,\mathrm{Hz}\times\alpha^{-1/6}\,. \tag{23}$$

This frequency falls well within the frequency range of future GW detectors such as LIGO A+, Voyager, Einstein telescope, cosmic explorer and DECIGO. The frequency (23) corresponds to an inter-PBH comoving separation of 600 km, which is also the mean separation between the remnants.

The PBH reheating scenario with PBH remnants has a unique prediction for the peak frequency of the induced GW spectrum. For a fixed PBH mass, the amplitude of the GW spectrum (15) only depends on β and its value today is given by

$$\Omega^{\mathrm{peak}}_{\mathrm{GW},0}h^2 \approx 10^{-11} r^{68/45}\alpha^{17/15}\left(\frac{\beta}{10^{-8}}\right)^{16/3}\,. \tag{24}$$

We plot the induced GW spectrum in figure 3. Recall that $1 > \alpha > 1/2$ and $r \sim O(1)$, so the predictions (22)–(24) do not depend much on whether PBHs have spin or if the remnant is a bit larger than the Planck length. Although this signal could also be present without remnants,

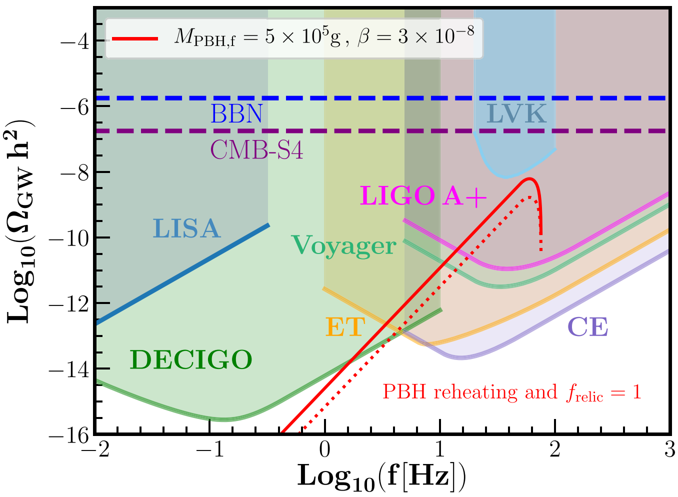

Figure 3. Spectral density of GWs induced by PBH number density fluctuations after PBH evaporation. The solid red line corresponds to the signal from the PBH reheating plus dark matter remnants scenario with $M_{\rm PBH,f} = 5 \times 10^5$ g and $\beta = 3 \times 10^{-8}$. The arbitrary choice of β corresponds to a peak amplitude $\Omega^{\rm peak}_{\rm GW,0} h^2 \sim 5 \times 10^{-9}$. The dashed red line is the same signal but for an almost extremal Kerr black hole, i.e. $\alpha = 1/2$. The amplitude of the signal only depends on β. The peak frequency is solely determined by the PBH mass which is fixed in this scenario. We also show for illustration the power-law integrated sensitivity curves [82] for LISA, DECIGO, Einstein telescope (ET), cosmic explorer (CE), voyager and LIGO A+ experiments. We used the sensitivity curves provided in [83–86]. In light blue we plot the upper bounds on the GW background from the LIGO/Virgo/KAGRA collaboration [87]. BBN and CMB data provide constraints on the energy density of any additional radiation, such as GWs. The horizontal long dashed lines qualitatively show the current constraint from BBN [60, 88, 89] (in blue) and future constraints from CMB-S4 experiments (in purple) [60, 90].

detecting a peak right at this frequency would be a strong indication that the PBH remnants is the DM.

3. Conclusions

The existence of remnants after Hawking evaporation is suggested in some theories of quantum gravity [11, 21, 22]. The remnants could play an important role in the information loss paradox and in cosmology [9]. Here we focused on the possibility that the Universe is filled with the remnants of tiny PBHs which evaporated well before BBN. For some parameter space the PBH remnants could account for all the DM and reheat the Universe [3, 18, 47].

One of the problems of the PBH reheating plus DM remnants scenario was that it seemed almost impossible to probe. In this note, we pointed out that it may lead to a unique prediction for the GW background: a peaked signal at a frequency ∼70 Hz, close to the peak sensitivity of LIGO/Virgo/KAGRA, LIGO A+, voyager Einstein telescope and cosmic explorer. The low frequency tail of the resulting GW spectrum would also be seen in DECIGO but it is out of reach for LISA. While this is not definitive evidence of remnants as DM, finding a peak at such a precise frequency would give a strong indication of the PBH reheating plus remnants scenario. This could be further probed by additional signatures of high frequency GWs and the effective number of species [49, 58, 60, 61, 91, 92].

A remaining issue is to derive a more accurate estimate for the amplitude of PBH isocurvature induced GWs, because the PBH number density fluctuations reach the nonlinear regime close to the final stage of evaporation [57, 63]. However, this requires sophisticated numerical simulations. Another issue is the effect of a finite width in the PBH mass function. While these issues might reduce the amplitude of the induced GW spectrum, the peak frequency would not be significantly affected. Thus the prediction for a GW background peaked at ∼70 Hz seems robust in the scenario of PBH remnants as DM within $O(1)$ factors.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Acknowledgment

We would like to thank the anonymous referee for the many useful comments. G D thanks A D Rojas, C D Rojas and D Rojas for their support and A Platania for useful correspondence on black holes in asymptotic safe gravity. We also thank P Bari and I Markin for useful comments on the paper. G D is supported by the DFG under the Emmy-Noether program Grant No. DO 2574/1-1, Project Number 496592360. Kavli IPMU is supported by World Premier International Research Center Initiative (WPI), MEXT, Japan. This work was supported in part by the JSPS KAKENHI Grant Nos. 19H01895, 20H04727, and 20H05853.

ORCID iDs

Guillem Domènech https://orcid.org/0000-0003-2788-884X
Misao Sasaki https://orcid.org/0000-0001-5924-0664

References

[1] Hawking S 1971 Gravitationally collapsed objects of very low mass *Mon. Not. R. Astron. Soc.* **152** 75
[2] Carr B J and Hawking S W 1974 Black holes in the early Universe *Mon. Not. R. Astron. Soc.* **168** 399–415
[3] MacGibbon J H 1987 Can Planck-mass relics of evaporating black holes close the Universe? *Nature* **329** 308–9
[4] Hawking S W 1974 Black hole explosions *Nature* **248** 30–31
[5] Hawking S W 1975 Particle creation by black holes *Commun. Math. Phys.* **43** 199–220
Hawking S W 1976 Particle creation by black holes *Commun. Math. Phys.* **46** 206 (erratum) (https://doi.org/10.1007/BF01608497)
[6] Manko V I and Markov M A 1977 Black holes evaporation and big mass particle (maximon, intermediate boson) creation in nonstationary Universe *Second Seminar on Quantum Gravity* (Springer) (available at: https://link.springer.com/book/10.1007/978-1-4613-2701-1#about-this-book)
[7] Hawking S W 1976 Breakdown of predictability in gravitational collapse *Phys. Rev.* D **14** 2460–73
[8] Susskind L 1995 Trouble for remnants (arXiv:hep-th/9501106)
[9] Chen P, Ong Y C and Yeom D-H 2015 Black hole remnants and the information loss paradox *Phys. Rep.* **603** 1–45

[10] Rovelli C and Vidotto F 2018 Small black/white hole stability and dark matter *Universe* **4** 127
[11] Ashtekar A and Bojowald M 2005 Black hole evaporation: a paradigm *Class. Quantum Grav.* **22** 3349–62
[12] Rovelli C and Vidotto F 2014 Planck stars *Int. J. Mod. Phys.* D **23** 1442026
[13] Bianchi E, Christodoulou M, D'Ambrosio F, Haggard H M and Rovelli C 2018 White holes as remnants: a surprising scenario for the end of a black hole *Class. Quantum Grav.* **35** 225003
[14] Nicolini P, Smailagic A and Spallucci E 2006 Noncommutative geometry inspired Schwarzschild black hole *Phys. Lett.* B **632** 547–51
[15] Chamseddine A H and Mukhanov V 2017 Nonsingular black hole *Eur. Phys. J.* C **77** 183
[16] Ben Achour J, Lamy F, Liu H and Noui K 2018 Non-singular black holes and the limiting curvature mechanism: a hamiltonian perspective *J. Cosmol. Astropart. Phys.* JCAP05(2018)072
[17] Chamseddine A H, Mukhanov V and Russ T B 2019 Black hole remnants *J. High Energy Phys.* JHEP10(2019)104
[18] Chen P and Adler R J 2003 Black hole remnants and dark matter *Nucl. Phys.* B **124** 103–6
[19] Hossenfelder S 2013 Minimal length scale scenarios for quantum gravity *Living Rev. Relativ.* **16** 2
[20] Vidotto F 2018 Quantum insights on primordial black holes as dark matter (arXiv:1811.08007)
[21] Eichhorn A and Held A 2022 Black holes in asymptotically safe gravity and beyond (arXiv:2212.09495 [gr-qc])
[22] Platania A 2023 Black holes in asymptotically safe gravity (arXiv:2302.04272 [gr-qc])
[23] Carr B J 1975 The primordial black hole mass spectrum *Astrophys. J.* **201** 1–19
[24] Khlopov M Y 2010 Primordial black holes *Res. Astron. Astrophys.* **10** 495–528
[25] Sasaki M, Suyama T, Tanaka T and Yokoyama S 2018 Primordial black holes—perspectives in gravitational wave astronomy *Class. Quantum Grav.* **35** 063001
[26] Green A M and Kavanagh B J 2021 Primordial black holes as a dark matter candidate *J. Phys. G: Nucl. Part. Phys.* **48** 043001
[27] Carr B, Kohri K, Sendouda Y and Yokoyama J 2021 Constraints on primordial black holes *Rep. Prog. Phys.* **84** 116902
[28] Carr B and Kuhnel F 2020 Primordial black holes as dark matter: recent developments *Annu. Rev. Nucl. Part. Sci.* **70** 355–94
[29] Escrivà A, Kuhnel F, and Tada Y 2022 Primordial black holes (arXiv:2211.05767 [astro-ph.CO])
[30] Kawasaki M, Kohri K and Sugiyama N 1999 Cosmological constraints on late time entropy production *Phys. Rev. Lett.* **82** 4168
[31] Kawasaki M, Kohri K and Sugiyama N 2000 MeV scale reheating temperature and thermalization of neutrino background *Phys. Rev.* D **62** 023506
[32] Hasegawa T, Hiroshima N, Kohri K, Hansen R S L, Tram T and Hannestad S 2019 MeV-scale reheating temperature and thermalization of oscillating neutrinos by radiative and hadronic decays of massive particles *J. Cosmol. Astropart. Phys.* JCAP12(2019)012
[33] Martin J, Papanikolaou T and Vennin V 2020 Primordial black holes from the preheating instability in single-field inflation *J. Cosmol. Astropart. Phys.* JCAP01(2020)024
[34] Animali C and Vennin V 2023 Primordial black holes from stochastic tunnelling *J. Cosmol. Astropart. Phys.* JCAP02(2023)043
[35] Briaud V and Vennin V 2023 Uphill inflation (arXiv:2301.09336 [astro-ph.CO])
[36] Carr B J 1976 Some cosmological consequences of primordial black-hole evaporations *Astrophys. J.* **206** 8–25
[37] Chapline G F 1976 Quarks in the early Universe *Nature* **261** 550–1
[38] Lidsey J E, Matos T and Urena-Lopez L A 2002 The inflaton field as selfinteracting dark matter in the brane world scenario *Phys. Rev.* D **66** 023514
[39] Hidalgo J C, Urena-Lopez L A and Liddle A R 2012 Unification models with reheating via primordial black holes *Phys. Rev.* D **85** 044055
[40] Turner M S 1979 Baryon production by primordial black holes *Phys. Lett.* B **89** 155–9
[41] Turner M S and Schramm D N 1979 The origin of baryons in the Universe and the astrophysical implications *Nature* **279** 303–5
[42] Zeldovich I B 1976 Charge asymmetry of the Universe as a consequence of evaporation of black holes and of the asymmetry of the weak interaction *ZhETF Pisma Redaktsiiu* **24** 29–32
[43] Zeldovich I B and Starobinskii A A 1976 Possibility of a cold cosmological singularity in the spectrum of primordial black holes *ZhETF Pisma Redaktsiiu* **24** 616–8
[44] Barrow J D, Copeland E J, Kolb E W and Liddle A R 1991 Baryogenesis in extended inflation. 2. Baryogenesis via primordial black holes *Phys. Rev.* D **43** 984–94

[45] Dolgov A D, Naselsky P D, and Novikov I D 2000 Gravitational waves, baryogenesis, and dark matter from primordial black holes (arXiv:astro-ph/0009407)
[46] Nagatani Y 2001 Electroweak domain wall by Hawking radiation: baryogenesis and dark matter from several hundred kg black holes (arXiv:hep-ph/0104160)
[47] Baumann D, Steinhardt P J, and Turok N 2007 Primordial black hole baryogenesis (arXiv:hep-th/0703250)
[48] Alexander S and Meszaros P 2007 Reheating, dark matter and baryon asymmetry: a triple coincidence in inflationary models (arXiv:hep-th/0703070)
[49] Fujita T, Kawasaki M, Harigaya K and Matsuda R 2014 Baryon asymmetry, dark matter and density perturbation from primordial black holes *Phys. Rev.* D **89** 103501
[50] Morrison L, Profumo S and Yu Y 2019 Melanopogenesis: dark matter of (almost) any mass and baryonic matter from the evaporation of primordial black holes weighing a ton (or less) *J. Cosmol. Astropart. Phys.* JCAP05(2019)005
[51] Datta S, Ghosal A and Samanta R 2021 Baryogenesis from ultralight primordial black holes and strong gravitational waves from cosmic strings *J. Cosmol. Astropart. Phys.* JCAP08(2021)021
[52] Smyth N, Santos-Olmsted L and Profumo S 2022 Gravitational baryogenesis and dark matter from light black holes *J. Cosmol. Astropart. Phys.* JCAP03(2022)013
[53] Anantua R, Easther R and Giblin J T 2009 GUT-scale primordial black holes: consequences and constraints *Phys. Rev. Lett.* **103** 111303
[54] Dolgov A D and Ejlli D 2011 Relic gravitational waves from light primordial black holes *Phys. Rev.* D **84** 024028
[55] Dong R, Kinney W H and Stojkovic D 2016 Gravitational wave production by Hawking radiation from rotating primordial black holes *J. Cosmol. Astropart. Phys.* JCAP10(2016)034
[56] Hooper D, Krnjaic G and McDermott S D 2019 Dark radiation and superheavy dark matter from black hole domination *J. High Energy Phys.* JHEP08(2019)001
[57] Inomata K, Kawasaki M, Mukaida K, Terada T and Yanagida T T 2020 Gravitational wave production right after a primordial black hole evaporation *Phys. Rev.* D **101** 123533
[58] Masina I 2020 Dark matter and dark radiation from evaporating primordial black holes *Eur. Phys. J. Plus* **135** 552
[59] Masina I 2021 Dark matter and dark radiation from evaporating kerr primordial black holes *Gravit. Cosmol.* **27** 315–30
[60] Arbey A, Auffinger J, Sandick P, Haghi B S E and Sinha K 2021 Precision calculation of dark radiation from spinning primordial black holes and early matter-dominated eras *Phys. Rev.* D **103** 123549
[61] Ireland A, Profumo S, and Scharnhorst J 2023 Primordial gravitational waves from black hole evaporation in standard and non-standard cosmologies (arXiv:2302.10188 [gr-qc])
[62] Papanikolaou T, Vennin V and Langlois D 2021 Gravitational waves from a Universe filled with primordial black holes *J. Cosmol. Astropart. Phys.* JCAP03(2021)053
[63] Domènech G, Lin C and Sasaki M 2021 Gravitational wave constraints on the primordial black hole dominated early Universe *J. Cosmol. Astropart. Phys.* JCAP04(2021)062
Domènech G, Lin C and Sasaki M 2021 Gravitational wave constraints on the primordial black hole dominated early Universe *J. Cosmol. Astropart. Phys.* JCAP11(2021)E01 (erratum) (https://doi.org/10.1088/1475-7516/2021/11/E01)
[64] Domènech G, Takhistov V and Sasaki M 2021 Exploring evaporating primordial black holes with gravitational waves *Phys. Lett.* B **823** 136722
[65] Domènech G 2021 Scalar induced gravitational waves review *Universe* **7** 398
[66] Samanta R and Urban F R 2022 Testing super heavy dark matter from primordial black holes with gravitational waves *J. Cosmol. Astropart. Phys.* JCAP06(2022)017
[67] Dalianis I and Kodaxis G P 2022 Reheating in runaway inflation models via the evaporation of mini primordial black holes *Galaxies* **10** 31
[68] Papanikolaou T 2022 Gravitational waves induced from primordial black hole fluctuations: the effect of an extended mass function *J. Cosmol. Astropart. Phys.* JCAP10(2022)089
[69] Kováčik S 2021 Hawking-radiation recoil of microscopic black holes *Phys. Dark Universe* **34** 100906
[70] Lehmann B V and Profumo S 2023 Black hole remnants are not too fast to be dark matter *Phys. Dark Universe* **39** 101145
[71] Di Gennaro S and Ong Y C 2021 Feasibility of primordial black hole remnants as dark matter in view of Hawking radiation recoil *J. Cosmol. Astropart. Phys.* JCAP07(2021)041

[72] Papanikolaou T 2023 Primordial black holes in loop quantum gravity: the effect on the threshold (arXiv:2301.11439 [gr-qc])
[73] Carr B J, Gilbert J H and Lidsey J E 1994 Black hole relics and inflation: limits on blue perturbation spectra *Phys. Rev.* D **50** 4853–67
[74] Iacconi L, Assadullahi H, Fasiello M and Wands D 2022 Revisiting small-scale fluctuations in α-attractor models of inflation *J. Cosmol. Astropart. Phys.* JCAP06(2022)007
[75] Aghanim N *et al* 2020 Planck 2018 results. VI. Cosmological parameters *Astron. Astrophys.* **641** A6
Aghanim N *et al* 2021 Planck 2018 results. VI. Cosmological parameters *Astron. Astrophys.* **652** C4 (erratum) (https://doi.org/10.1051/0004-6361/201833910e)
[76] Arbey A, Auffinger J and Silk J 2020 Evolution of primordial black hole spin due to Hawking radiation *Mon. Not. R. Astron. Soc.* **494** 1257–62
[77] Mukhanov V 2005 *Physical Foundations of Cosmology* (Cambridge University Press)
[78] Saikawa K and Shirai S 2018 Primordial gravitational waves, precisely: the role of thermodynamics in the standard model *J. Cosmol. Astropart. Phys.* JCAP05(2018)035
[79] Domènech G, Passaglia S and Renaux-Petel S 2022 Gravitational waves from dark matter isocurvature *J. Cosmol. Astropart. Phys.* JCAP03(2022)023
[80] Inomata K, Kohri K, Nakama T and Terada T 2019 Enhancement of gravitational waves induced by scalar perturbations due to a sudden transition from an early matter era to the radiation era *Phys. Rev.* D **100** 043532
[81] Inomata K, Kohri K, Nakama T and Terada T 2019 Gravitational waves induced by scalar perturbations during a gradual transition from an early matter era to the radiation era *J. Cosmol. Astropart. Phys.* JCAP10(2019)071
[82] Thrane E and Romano J D 2013 Sensitivity curves for searches for gravitational-wave backgrounds *Phys. Rev.* D **88** 124032
[83] Cosmic explorer sensitivity curve 2023 (available at: https://cosmicexplorer.org/sensitivity.html) (Accessed 5 May 2023)
[84] The A+ design curve 2023 (available at: https://dcc.ligo.org/LIGO-T1800042/public) (Accessed 5 May 2023)
[85] Ligo unofficial sensitivity curves 2023 (available at: https://dcc.ligo.org/LIGO-T1500293/public) (Accessed 5 May 2023)
[86] Schmitz K 2021 New sensitivity curves for gravitational-wave signals from cosmological phase transitions *J. High Energy Phys.* JHEP01(2021)097
[87] Abbott R *et al* 2021 Upper limits on the isotropic gravitational-wave background from Advanced LIGO and advanced Virgo's third observing run *Phys. Rev.* D **104** 022004
[88] Cyburt R H, Fields B D, Olive K A and Skillman E 2005 New BBN limits on physics beyond the standard model from ^{4}*He Astropart. Phys.* **23** 313–23
[89] Grohs E and Fuller G M 2023 Big bang nucleosynthesis (arXiv:2301.12299 [astro-ph.CO])
[90] Abazajian K N *et al* 2016 CMB-S4 science book, 1st edn (arXiv:1610.02743 [astro-ph.CO])
[91] Hooper D, Krnjaic G, March-Russell J, McDermott S D, and Petrossian-Byrne R 2020 Hot gravitons and gravitational waves from kerr black holes in the early Universe (arXiv:2004.00618 [astro-ph.CO])
[92] Kazemian S, Pascual M, Rovelli C and Vidotto F 2023 Diffuse emission from black hole remnants *Class. Quantum Grav.* **40** 087001