Symmetries for Interval Analysis

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Introduction

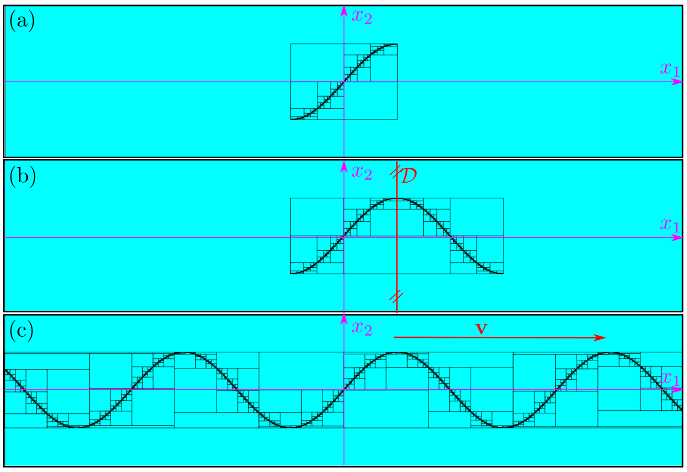
Interval analysis relies on a catalog of basic constraints such as

$$\begin{array}{ll}
(i) & x_1 + x_2 = x_3 \\
(ii) & x_1 \cdot x_2 = x_3 \\
(iii) & x_2 = x_1^2 \\
(iv) & x_2 = \sin(x_1) \\
(v) & \left(\begin{array}{c} x_1 \\ x_2 \end{array}\right) = \left(\begin{array}{cc} x_3 & x_4 \\ x_5 & x_6 \end{array}\right) \cdot \left(\begin{array}{c} x_7 \\ x_8 \end{array}\right) \\
(vi) & \left(\begin{array}{c} x_1 \\ x_2 \end{array}\right) = \left(\begin{array}{c} x_3 \cdot \cos(x_4) \\ x_3 \cdot \sin(x_4) \end{array}\right)
\end{array} \quad (1)$$

For each of these constraints, we have to build minimal contractors. To solve a problem defined by nonlinear constraints [1], an interval solver decomposes it into constraints that are inside the catalog. Then, it calls the associated contractors until no more contractions can be observed.

Minimal contractors

Denote by $\mathbb{IR}^n$ the set of boxes of $\mathbb{R}^n$. A *minimal contractor* $\mathcal{C}^*$ for a constraint can be defined as an operator from $\mathbb{IR}^n$ to $\mathbb{IR}^n$ such that $\mathcal{C}^*([\mathbf{x}])$ corresponds to the smallest box which can be obtained by a contraction of $[\mathbf{x}]$ without removing a single point of the constraint. Now, many constraints such as those in (1) can be generated by applying specific symmetries (translation, hyperoctahedral, scaling, ...) [2] to a monotonic constraint. This will allow us to build efficient optimal contractors for a large class of constraints. The principle is illustrated by the figure below for the constraint (iv) where (a) represents the generator, (b) the action of the axial symmetry $\mathcal{D}$ and (c) the action of the translation symmetry $\mathbf{v}$.



In the presentation we will consider much complex constraints related to localization problems.

References

[1] S. Rohou, L. Jaulin, L. Mihaylova, F. Le Bars and S. Veres, *Reliable robot localization*, ISTE group, 2019.

[2] B. Desrochers and L. Jaulin, A Minimal Contractor for the Polar Equation; Application to Robot Localization, *Engineering Applications of Artificial Intelligence*, 2016.