A Refinement of the Millionshchikov Quasi-Normality Hypothesis for Convective Boundary Layer Turbulence

VLADIMIR M. GRYANIK

Alfred Wegener Institute for Polar and Marine Research, Bremerhaven, Germany, and A. M. Obukhov Institute of Atmospheric Physics, Russian Academy of Sciences, Moscow, Russia

JÖRG HARTMANN

Alfred Wegener Institute for Polar and Marine Research, Bremerhaven, Germany

SIEGFRIED RAASCH AND MICHAEL SCHRÖTER

University of Hannover, Hannover, Germany

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ABSTRACT

The Millionshchikov hypothesis of quasi-normal distribution of fourth-order moments fails for convective conditions where the probability density functions of temperature and vertical velocity fluctuations are skewed. This is shown for aircraft and large-eddy simulation (LES) data, and new closures for fourth-order moments that take the skewness into account are suggested. These new closures are in very good agreement with the data.

1. Introduction

The Millionshchikov hypothesis (Millionshchikov 1941; Monin and Yaglom 1971) states that in higher-order turbulence closure (HOC) models, the fourth-order moments (FOM) can be approximated as quasi-normal (Gaussian) even if the third-order moments (TOM) are nonzero. For a long time, this hypothesis was adopted without discussion in a large number of turbulence closure models (e.g., Andre et al. 1976; Moeng and Randall 1984; Canuto et al. 1994). Recent theoretical studies (e.g., Ilyushin and Kurbatskii 1997; Canuto et al. 2001; Gryanik and Hartmann 2002, hereafter GH) and analyses of measurements (e.g., GH; Alberghi et al. 2002) indicate that the Millionshchikov hypothesis fails for vertical velocity and temperature in convective boundary layer (CBL) conditions. Furthermore, we have for the first time FOMs from large-eddy simulation (LES) data available. They also show that the Millionshchikov hypothesis fails for the same con-

ditions. Losch (2004) found very similar behavior of the FOMS in open-ocean deep convection. In this note, we present a refinement of the Millionshchikov hypothesis as applied to CBL conditions.

Using aircraft measurements and LES data we show that the FOM in vertical velocity and temperature are essentially non-Gaussian, while the FOM in the horizontal along and across wind components are close to Gaussian.

In GH, we explained the shortcoming of the Millionshchikov hypothesis by the skewed nature of CBL turbulence with respect to upward–downward and hot–cold fluctuations and derived non-Gaussian FOM parameterizations for $\overline{w'^4}$, $\overline{w'^3\theta'}$, $\overline{w'\theta'^3}$, and $\overline{\theta'^4}$ (here, w' is the vertical velocity fluctuation and θ' the temperature fluctuations), which explicitly account for the effects of asymmetry between upward and downward motion and also between warm and cold areas of the fluid by semi-organized coherent structures. Here we extend the two-scale top-hat method of GH to also include the horizontal velocity components leading to a 16-delta model of the probability density function (PDF). We test these parameterizations of GH against a new set of LES data, and suggest new non-Gaussian FOM parameterizations for the other moments not yet considered but explicitly

Corresponding author address: Dr. Jörg Hartmann, Alfred Wegener Institute for Polar and Marine Research, Postfach 12 0161, Bremerhaven 27515, Germany.
E-mail: jhartmann@awi-bremerhaven.de

involved into HOC models $\overline{w'^2\theta'^2}$, $\overline{u'^4}$, $\overline{v'^4}$, $\overline{w'^2u'^2}$, $\overline{w'^2v'^2}$, $\overline{w'^2q'^2}$ (here, u', v' are the along-wind and cross-wind horizontal velocity components, $q'^2 = u'^2 + v'^2 + w'^2$). For all the moments considered, the agreement between the parameterizations proposed and the data is very good.

2. Testing of the Millionshchikov hypothesis versus data and LES

The aircraft measurements are obtained during the Arctic Radiation and Turbulence Interaction Study (ARTIST) campaign (Hartmann et al. 1999) in a convectively driven boundary layer over the ocean with 8–12 m s^{-1} wind speed, further details can also be found in GH. The LES data are from a simulation of a convective boundary layer with 12 m s^{-1} geostrophic wind speed, they are described in detail in Raasch and Schröter (2001). In Fig. 1 the fourth order moments are plotted versus their Gaussian parameterizations (Monin and Yaglom 1971)

$$\overline{a'b'c'd'} = \overline{a'b'} \cdot \overline{c'd'} + \overline{a'c'} \cdot \overline{b'd'} + \overline{a'd'} \cdot \overline{b'c'}, \quad (1)$$

where a', b', c', d' are the fluctuations w', u', v', and θ'. Since the horizontal velocity components have a very small skewness, the FOM of the along-wind $\overline{u'^4}$ and cross-wind $\overline{v'^4}$ horizontal velocities agree with the Millionshchikov hypothesis both for measurements and LES data.

All fourth-order moments involving the vertical velocity component or the temperature mostly are significantly larger than their Gaussian prediction. The Gaussian prediction gives an estimation of a lower bound for all these moments, except for the FOM of the across-wind component $\overline{v'^4}$. As an indication for the suitability of the parameterization we calculate the explained variances $\sigma_f^2 = 1 - \overline{(y_i - f(x_i))^2}/\overline{(y_i - \bar{y})^2}$, where y_i are the aircraft measurements and LES data, plotted at the ordinate, and $f(x_i)$ the parameterizations (abscissas). For the vertical wind component of the LES data, the Gaussian parameterization $3\overline{w'^2}^2$ only explains 68% of the variance of $\overline{w'^4}$, and for the aircraft data 67%. The agreement is worse for some cross moments for the aircraft data since measurement accuracy also contributes to the unexplained variance, see also GH. The LES data do not suffer such errors and lead to higher values of the explained variance. The LES data are plotted as a continuous line in the order of increasing height. The loops in these plots, especially for the moments $\overline{w'^4}$, $\overline{w'^3\theta'}$ and $\overline{w'^2\theta'^2}$, represent an ambiguity since the relation is not constant with height. An ambiguity like this is a serious drawback of any parameterization.

3. New expressions for FOM

Gryanik and Hartmann (2002) assumed that a measure of deviation of statistics from the Gaussian behavior is provided by the skewness ($S_x = (\overline{x'^3})/[\overline{x'^2}^{(3/2)}]$) of vertical velocity S_w and temperature S_θ, and suggested the parameterizations

$$\overline{w'^4} = a_3(1 + d_3 S_w^2)\overline{w'^2}^2, \quad (2)$$

$$\overline{\theta'^4} = a_4(1 + d_4 S_\theta^2)\overline{\theta'^2}^2, \quad (3)$$

$$\overline{w'^3\theta'} = a_5(1 + d_5 S_w^2)\overline{w'^2}\,\overline{w'\theta'}$$

$$= a_5(1 + d_5 S_w^2)C_{w\theta}\overline{w'^2}^{3/2}\overline{\theta'^2}^{1/2}, \quad (4)$$

$$\overline{w'\theta'^3} = a_6(1 + d_6 S_\theta^2)\overline{\theta'^2}\,\overline{w'\theta'}$$

$$= a_6(1 + d_6 S_\theta^2)C_{w\theta}\overline{w'^2}^{1/2}\overline{\theta'^2}^{3/2}, \quad (5)$$

where

$$C_{w\theta} = \frac{\overline{w'\theta'}}{\overline{w'^2}^{1/2}\overline{\theta'^2}^{1/2}} \quad (6)$$

is the correlation coefficients of w and θ, and a_i ($i = 3, \ldots, 11$) and d_i ($i = 3, \ldots, 11$) are dimensionless parameters. Equations (2)–(5) are discussed in GH in detail and will here be tested against the new LES data of low-to-moderate wind.

If we, in analogy to GH, assume the skewness of the along-wind S_u and cross-wind S_v horizontal velocity components to be a measure of the deviation of statistics from the Gaussian behavior, we suggest generalizing the Millionshchikov hypothesis to the new FOM closure of the following form:

$$\overline{w'^2\theta'^2} = a_7[(\overline{w'^2}\,\overline{\theta'^2} + 2\overline{w'\theta'}^2) + d_7 S_w S_\theta \overline{w'\theta'}\,\overline{w'^2}^{1/2}\overline{\theta'^2}^{1/2}]$$

$$= a_7\left(1 + d_7\frac{C_{w\theta}}{1 + 2C_{w\theta}^2}S_w S_\theta\right)(\overline{w'^2}\,\overline{\theta'^2} + 2\overline{w'\theta'}^2), \quad (7)$$

$$\overline{u'^4} = a_8(1 + d_8 S_u^2)\overline{u'^2}^2, \quad (8)$$

$$\overline{v'^4} = a_9(1 + d_9 S_v^2)\overline{v'^2}^2, \quad (9)$$

$$\overline{w'^2u'^2} = a_{10}\left(1 + d_{10}\frac{C_{wu}}{1 + 2C_{wu}^2}S_w S_u\right)$$

$$\times (\overline{w'^2}\,\overline{u'^2} + 2\overline{w'u'}^2), \quad (10)$$

$$\overline{w'^2v'^2} = a_{11}\left(1 + d_{11}\frac{C_{wv}}{1 + 2C_{wv}^2}S_w S_v\right)(\overline{w'^2}\,\overline{v'^2} + 2\overline{w'v'}^2), \quad (11)$$
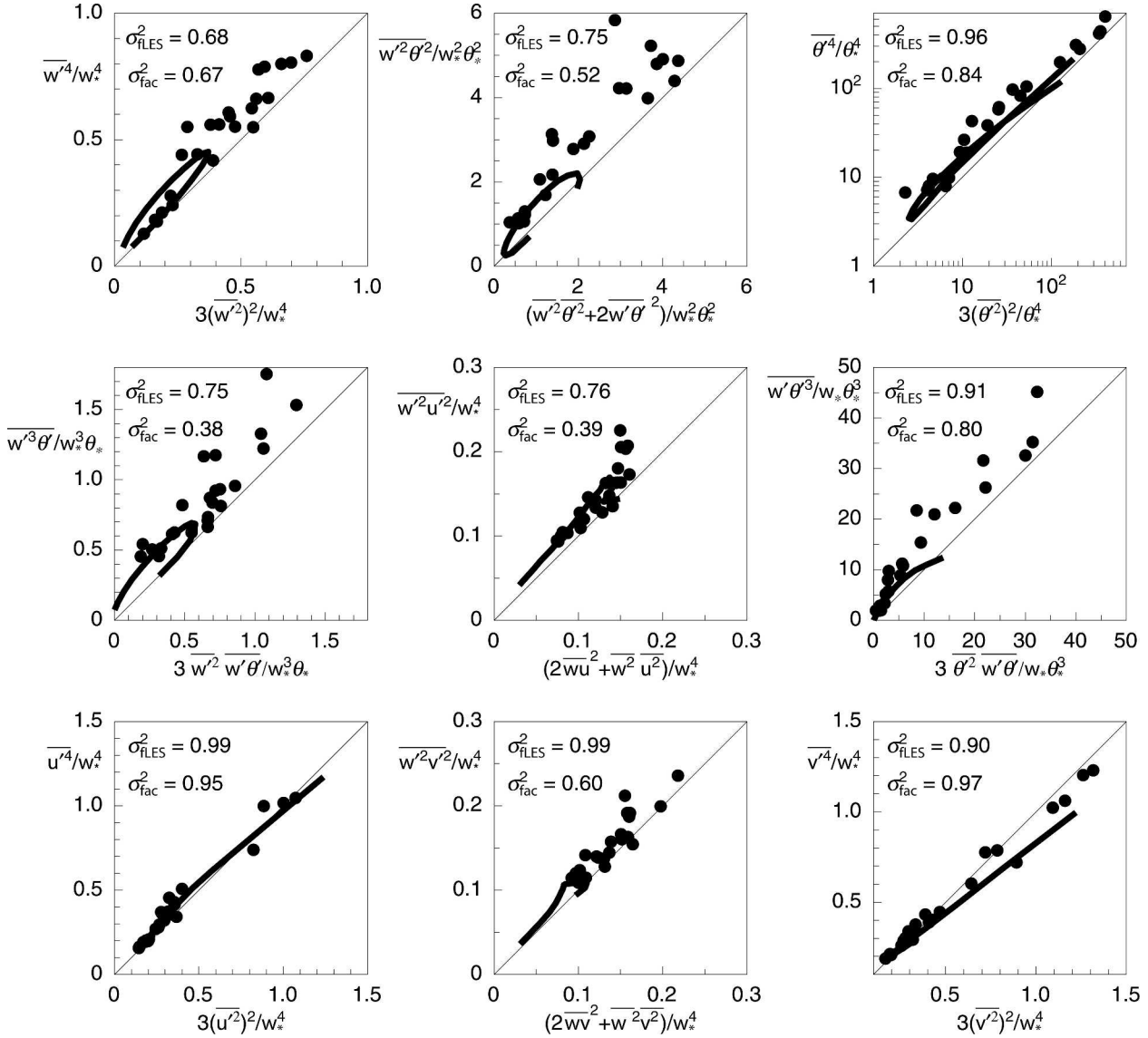
FIG. 1. Aircraft measurements (dots) and LES results (solid lines) of fourth-order moments (ordinates) vs their parameterizations (abscissas) based on the Gaussian assumption. Since the LES data do not include subgrid scales, the range of values for some moments is smaller than that of the aircraft data, especially where large values occur near the surface and near the inversion. Deardorff scaling with θ_* and w_* is used for normalization and σ_f are the explained variances. The Gaussian parameterization systematically underestimates the FOMs. Furthermore, it leads to an ambiguity of the relation between moment and parameterization for different heights in the boundary layer. This is especially seen in the loop feature of the LES results, which are plotted as a continuous line for increasing height.

where

$$C_{wu} = \frac{\overline{w'u'}}{\overline{w'^2}^{1/2}\,\overline{u'^2}^{1/2}}, \quad C_{wv} = \frac{\overline{w'v'}}{\overline{w'^2}^{1/2}\,\overline{v'^2}^{1/2}} \quad (12)$$

are correlation coefficients. The moments (10) and (11) can be formulated in analogy to the first line in (7).

This choice fulfills the conditions given by (i) dimensional analysis, (ii) tensor invariance, (iii) symmetry conditions (Mironov et al. 1999; Abdella and McFarlane 1999), (iv) realizability requirements (Andre et al. 1976), and (v) solution for the 16-delta probability density function model [which is a straightforward generalization of the 3-delta PDF model of Mironov et al. (1999) and GH and the traditional 2-delta PDF model, e.g., Lappen and Randall (2001), see Eq. (13) below].

The parameterizations (2)–(11) shall represent the traditional Millionshchikov hypothesis in the case of

zero skewness. In the limit of large skewness they shall be derived from an analysis of joint probabilities. We approximate the actual PDF by a simplification that represents the contribution of the coherent structures. In this model, all deviations from the mean of vertical velocity w, potential temperature θ, along-stream velocity u, and cross-stream velocity v are represented by eight values: w_u and w_d updraft and downdraft velocities, θ_w and θ_c warm and cold fluctuations, u_f and u_b forward and backward velocities, and v_r and v_l left and right velocities. In analogy to a mass-flux representation we can form joint probabilities for these eight quantities: p_{uwfr}, p_{dcfr} are the joint probabilities of warm

updraft w_u, θ_w and cold downdraft w_d, θ_c together with the joint probability p_{ucfr}, p_{dwfr} of cold updraft w_u, θ_c and warm downdraft w_d, θ_w in along-wind forward component u_f with deviation to the right v_r. The joint probabilities p_{uwfl}, p_{dcfl}, p_{ucfl}, p_{dwfl} are the probabilities of warm updraft, cold downdraft, cold updraft, and warm downdraft of the along-wind forward component with deviation to the left v_l. The remaining joint probabilities p_{uwbr}, p_{dcbr}, p_{ucbr}, p_{dwbr}, and p_{uwbl}, p_{dcbl}, p_{ucbl}, p_{dwbl} of along-wind backward streams u_b with deviations to the right v_r and to the left v_l, respectively, are defined in the same way.

The 16-delta PDF can be formulated by

$$
\begin{aligned}
P(w', \theta', u', v') = &[p_{\mathrm{uwfr}}\delta(w' - w_u)\delta(\theta' - \theta_w) + p_{\mathrm{dcfr}}\delta(w' - w_d)\delta(\theta' - \theta_c) + p_{\mathrm{ucfr}}\delta(w' - w_u)\delta(\theta' - \theta_c) \\
&+ p_{\mathrm{dwfr}}\delta(w' - w_d)\delta(\theta' - \theta_w)]\delta(u' - u_f)\delta(v' - v_r) + [p_{\mathrm{uwbr}}\delta(w' - w_u)\delta(\theta' - \theta_w) \\
&+ p_{\mathrm{dcbr}}\delta(w' - w_d)\delta(\theta' - \theta_c) + p_{\mathrm{ucbr}}\delta(w' - w_u)\delta(\theta' - \theta_c) \\
&+ p_{\mathrm{dwbr}}\delta(w' - w_d)\delta(\theta' - \theta_w)]\delta(u' - u_b)\delta(v' - v_r) + [p_{\mathrm{uwfl}}\delta(w' - w_u)\delta(\theta' - \theta_w) \\
&+ p_{\mathrm{dcfl}}\delta(w' - w_d)\delta(\theta' - \theta_c) + p_{\mathrm{ucfl}}\delta(w' - w_u)\delta(\theta' - \theta_c) \\
&+ p_{\mathrm{dwfl}}\delta(w' - w_d)\delta(\theta' - \theta_w)]\delta(u' - u_f)\delta(v' - v_l) + [p_{\mathrm{uwbl}}\delta(w' - w_u)\delta(\theta' - \theta_w) \\
&+ p_{\mathrm{dcbl}}\delta(w' - w_d)\delta(\theta' - \theta_c) + p_{\mathrm{ucbl}}\delta(w' - w_u)\delta(\theta' - \theta_c) \\
&+ p_{\mathrm{dwbl}}\delta(w' - w_d)\delta(\theta' - \theta_w)]\delta(u' - u_b)\delta(v' - v_l),
\end{aligned}
\tag{13}
$$

where $\delta(x)$ is the Dirac delta function.

If we integrate the PDF (13) over the horizontal wind components, the 16-delta model coincides with the 4-delta (4-quadrant) mass-flux model of updrafts/downdrafts and warm/cold areas, where

$$p_{\mathrm{uwfr}} + p_{\mathrm{uwbr}} + p_{\mathrm{uwfl}} + p_{\mathrm{uwbl}} = p_{\mathrm{uw}}$$

the probability of warm updraft, (14)

$$p_{\mathrm{ucfr}} + p_{\mathrm{uebr}} + p_{\mathrm{ucfl}} + p_{\mathrm{uebl}} = p_{\mathrm{ue}}$$

the probability of cold updraft, (15)

$$p_{\mathrm{dwfr}} + p_{\mathrm{dwbr}} + p_{\mathrm{dwfl}} + p_{\mathrm{dwbl}} = p_{\mathrm{dw}}$$

the probability of warm downdraft, (16)

$$p_{\mathrm{dcfr}} + p_{\mathrm{dcbr}} + p_{\mathrm{dcfl}} + p_{\mathrm{dcbl}} = p_{\mathrm{dc}}$$

the probability of cold downdraft. (17)

If we assume further $p_{\mathrm{dw}} = 0$, the 3-delta model of Mironov et al. (1999) is recovered. The further simplification $p_{\mathrm{uc}} = 0$ leads to the traditional 2-delta mass-flux model, (e.g., Zilitinkevich et al. 1999; Lappen and Randall 2001).

By comparison with the Millionshchikov assumption

[Eq. (1)] the first requirement (Gaussian behavior in the limit of zero skewness) leads to

$$a_i = 3(i = 3, \ldots, 6, 8, 9), \quad a_i = 1(i = 7, 10, 11). \tag{18}$$

By straightforward algebraic calculation of the moment with the help of the PDF (13) we find the moments in terms of PDF parameters. Next we solve the lowest order equations with respect to the parameters and substitute the result into the equations for the higher order moments, see GH for details [their Eqs. (4)–(17)]. We find in the limit of large skewness

$$\overline{w'^4} = S_w^2 \overline{w'^2}^2 \tag{19}$$

$$\overline{w'^3\theta'} = S_w^2 \overline{w'\theta'}\, \overline{w'^2}, \tag{20}$$

$$\overline{w'^2\theta'^2} = S_w S_\theta \overline{w'\theta'}\, \overline{w'^2}^{1/2}\overline{\theta'^2}^{1/2}, \tag{21}$$

$$\overline{w'\theta'^3} = S_\theta^2 \overline{w'\theta'}\, \overline{\theta'^2}, \tag{22}$$

$$\overline{\theta'^4} = S_\theta^2 \overline{\theta'^2}^2. \tag{23}$$

In analogy, the moments involving the horizontal wind components u and v are formed. Comparing (19)–(23) with (2)–(7) and the corresponding equations for the u
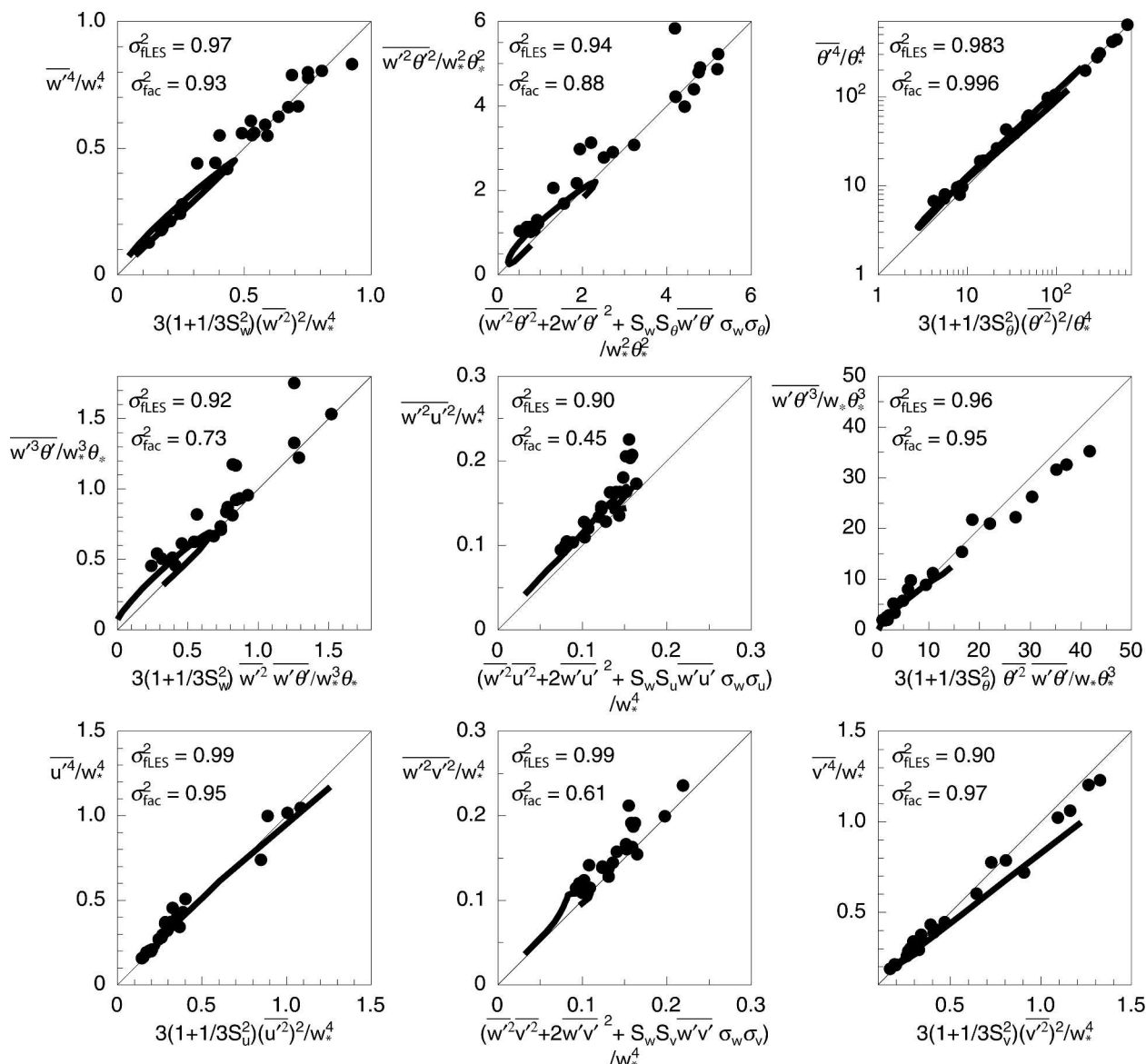
FIG. 2. Aircraft measurements (dots) and LES results (solid lines) of fourth-order moments (ordinates) vs the new parameterizations Eqs. (2)–(11) (abscissas); σ_f are the explained variances. Since the LES data do not include subgrid scales, the range of values for some moments is smaller than that of the aircraft data, especially where large values occur near the surface and near the inversion.

and v components with (8)–(11), we find in the limit of large skewness

$$d_i = \frac{1}{3} \, (i = 3, \ldots, 8, 9), \quad d_i = 1(i = 7, 10, 11). \quad (24)$$

For $i = 3, \ldots, 6$, Eqs. (18) and (24) coincide with the coefficients obtained for the 3-delta PDF model in GH. Substitution of the nondimensional coefficients a_i (18) and d_i (24) into Eqs. (2)–(12) gives the universal closure model, which describes the Gaussian convective regime in the limit of small skewness, and strongly non-

Gaussian skewed turbulence in the limit of large skewness. It presents a linear interpolation between these two limiting conditions.

4. Testing of new FOM versus data and LES

In Fig. 2 we compare the refined Millionshchikov parameterizations given by Eqs. (2)–(11) versus measurements and LES data. Based on the explained variance, all moments show a better agreement for both aircraft measurements and LES data, except the fourth-

order moment of the across-wind component $\overline{v'^4}$, which is virtually unchanged. Furthermore, the ambiguity in the relation for different heights has almost disappeared, seen as the nearly collapsed loop in the LES data. The parameterization becomes single valued for the moments $\overline{w'^4}$, $\overline{\theta'^4}$ and $\overline{w'\theta'^3}$, and reduces for $\overline{w'^3\theta'}$ of the LES results.

We also tested the parameterizations (2)–(11) with LES simulation data of 4 and 0 m s^{-1} geostrophic wind speed while all other boundary layer parameters were unchanged. These results also confirm the unsuitability of the Millionshchikov hypothesis and show a significantly better agreement with our new formulations.

5. Realizability of the new parameterizations

The drawback of the Millionshchikov closure for convective conditions becomes also evident from the analysis of measurements and LES data. The analysis shows that statistics (SOM, TOM, and FOM) do not satisfy the realizability conditions (Andre et al. 1976)

$$\overline{a'b'c'}^2 \leq \min \begin{cases} \overline{a'^2}(\overline{b'^2c'^2} - \overline{b'^2} \cdot \overline{c'^2}), \\ \overline{b'^2}(\overline{a'^2c'^2} - \overline{a'^2} \cdot \overline{c'^2}), \\ \overline{c'^2}(\overline{a'^2b'^2} - \overline{a'^2} \cdot \overline{b'^2}), \end{cases} \quad (25)$$

where a', b', c', d' are arbitrary fluctuations of u', v', w', θ'.

The measurements and LES statistics satisfy the realizability conditions (25) as shown for the potential temperature θ in Fig. 3, which shows the realizability condition for $a = b = c = \theta$

$$K_\theta \geq 1 + S_\theta^2, \quad (26)$$

where $K_\theta = \overline{\theta'^4}/\overline{\theta'^2}^2$ is the kurtosis and S_θ the skewness. If the kurtosis is calculated by the Millionshchikov closure hypothesis (1) we have $K_\theta = 3$ (plotted by the horizontal line in Fig. 3), the realizability condition (26) would be violated for $S_\theta > \sqrt{2}$. The closure (3) is in agreement with condition (26), since the $K_\theta = 3(1 + \frac{1}{3}S_\theta^2) \geq 1 + S_\theta^2$. The other realizability conditions, for example, $K_\theta \geq 1 + \overline{w'\theta'^2}^2/(\overline{w'^2}\,\overline{\theta'^2}^2)$ following from (25) for $a = b = \theta$ and $c = w$ are also satisfied by the new FOM parameterizations. Substituting the closure (3) into the lhs, we find $S_\theta^2 \geq \overline{w'\theta'^2}^2/(\overline{w'^2}\,\overline{\theta'^2}) - 2$. This condition is satisfied since $S_\theta^2 \geq 0$ and $\overline{w'\theta'^2}^2/(\overline{w'^2}\,\overline{\theta'^2}) - 2 \leq -1$.

6. Discussion and conclusions

The measurements and LES data provide counterexamples for the Millionshchikov hypothesis. The results
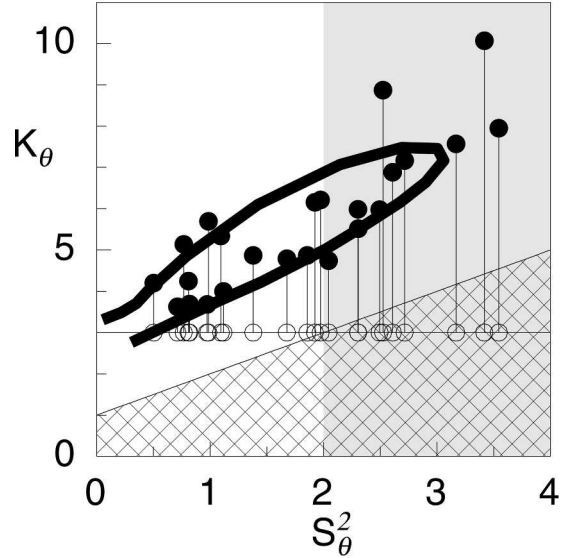

FIG. 3. Kurtosis vs the square of the skewness for the aircraft data (dots) and the LES results (solid lines). Hatching marks the nonrealizable area. The data with $S^2 > 2$ (light gray area) are not realizable with the Gaussian parameterization.

presented in Figs. 1 and 2 show that overall at CBL conditions, the Millionshchikov hypothesis is not well suited for parameterization of FOM involving the vertical velocity and temperature.

In the range of parameters studied, the parameterizations involving horizontal velocity fluctuations have little skewness and are close to Gaussian. Here, the Millionshchikov hypothesis remains valid for these moments, and the simpler parameterization (1) may be used.

Implementation of the parameterizations (2)–(11) into the transport terms of the TOM equations shows that the redistributions of the FOMs not only depend on the gradients of the SOMs, as the Millionshchikov hypothesis states, but also on the gradients of the TOMs as well.

Since the parameterizations (2)–(11) express the FOM in terms of second-order moments and TOM, the minimal closure model for convective conditions should be the TOM closure model.

The TOM model can be simplified, for example, assuming stationarity of the TOM equations (cf. Canuto and Dubovikov 1998). It can be reduced to a simple SOM equations model.

The universal model may also be valid for moisture and passive scalar fluctuations, which shall be examined in further studies.

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